UNIT - I

Basic Circuit Analysis

Electric Charge:-

Charge is an electric property of atomic particle, whose unit is Coulombs (C). Atom consists of electron, protons and neutrons.

electrons - negative charge

protons - positive charge

Neutrons - equal potential of + & -

1 Coulombs of charge = 6.28×10^{18} electrons.

Charge of 1 electron = -1.602×10^{-19} coulombs

symbol of charge = Q.

Electric Current:-

It is defined as rate of flow of charge in an electric circuit (or) flow of electrons.

$$I = \frac{dQ}{dt}$$, unit - Ampere (A).

Electric Potential:-

The ability of a charged particle to do a work is called electric potential.

$$\text{Electric potential} = \frac{\text{Workdone}}{\text{Charge}} = \frac{W}{Q}$$

Potential Difference :-

The workdone to move a unit positive (or) negative charge from one point to another point

It is measured in volts (V).

Electric Work :-

The workdone to move a charge of 1 coulombs from one point to another point with a potential difference 1 Volt.

$$\underset{(W)}{\text{Electric work}} = V \times Q$$

$$I = \frac{Q}{t}$$

$$\therefore \boxed{W = V.I.t} \text{ Joules}$$

Electrical workdone is nothing but electrical energy.

Electric power :-

The rate at which electrical work is done in an electric circuit is called an electrical power.

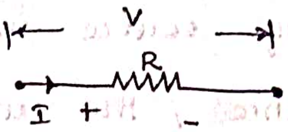
$$\text{Electrical power } (P) = \frac{\text{Workdone}}{\text{Time}} = \frac{W}{t}$$

$$= \frac{V.I.t}{t}$$

$$\boxed{P = V.I \quad \text{Joule/sec or Watts}}$$

Electric Circuit Elements

(i) Resistance :-



It is the property of a material, by which it opposes the flow of current through it.

It is denoted by 'R'. unit is 'Ω'.

$$R = \frac{\rho l}{A}$$

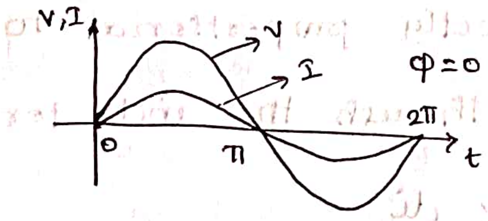
$\rho \rightarrow$ resistivity of the material

$l \rightarrow$ length of the material

$A \rightarrow$ Area of the material.

$R \rightarrow$ resistance of the conductor.

When we apply AC Voltage and AC current through a resistance, both will be in phase.



(ii) Capacitance :-

It is an element which stores the applied energy in the form of electrostatic field. It can be denoted by 'C'. unit is 'F'.

$$V = \frac{1}{C}\int i\,dt$$

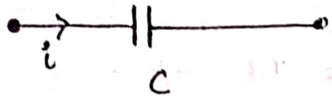
or

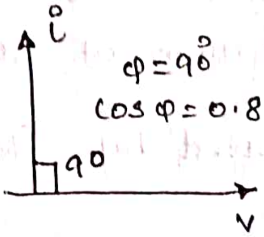
$$\frac{dv}{dt} = \frac{1}{C} \times i \Rightarrow \boxed{i = C.\frac{dv}{dt}}$$

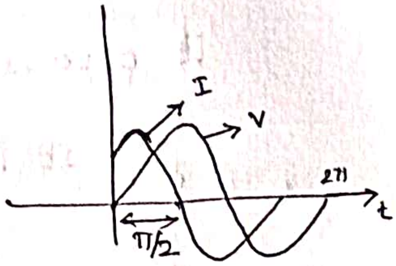
Energy stored in capacitor,

$$E = \frac{1}{2} CV^2 \text{ Joules}$$

when we apply ac voltage source and ac current through pure capacitance, the current will lead the voltage by 90°.







Average power consumed by pure capacitance $\Big\}$ $P_{avg} = 0$.

Inductance :-

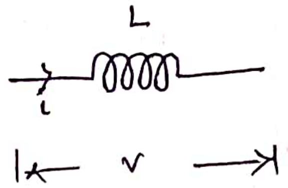
It is the property of a material by which electrical energy stored in the form of electromagnetic field. It is denoted by 'L', unit is 'H'. The voltage across the inductor is directly proportional to the rate of change of current through the inductor.

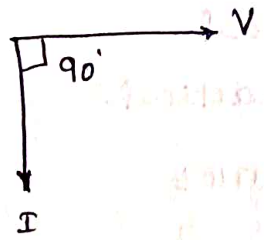
$$V \propto \frac{di}{dt}$$

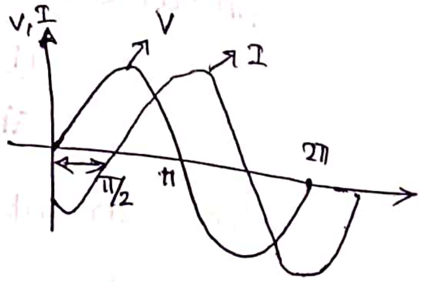
$$V = L \cdot \frac{di}{dt}$$

Energy stored in inductor $E = \frac{1}{2} L i^2$ Joules.

When an ac voltage & current is applied to a pure inductance, the current will be lag the voltage by 90°.







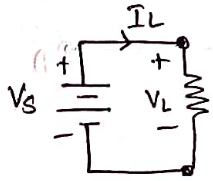
The avg power consumed by pure inductance is Zero.

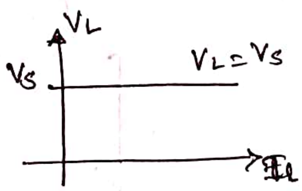
Circuit Elements	Voltage Equation	Current Equation	Power Equation	Energy Equation.
R	$V = IR$	$I = \frac{V}{R}$	$P = VI$	$E = V.I.t$
L	$V = L.\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt$	$P = v.i$ $= L.i.\frac{di}{dt}$	$E = \frac{1}{2}Li^2$
C	$V = \frac{1}{c}\int i\,dt$	$i = C\frac{dv}{dt}$	$P = v.i$ $= C.V.\frac{dv}{dt}$	$E = \frac{1}{2}CV^2.$

Active Elements:-

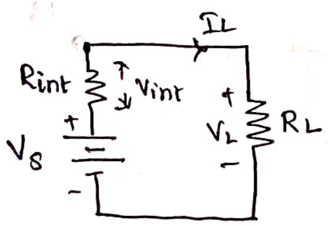
(i) voltage source
- ideal
- practical.

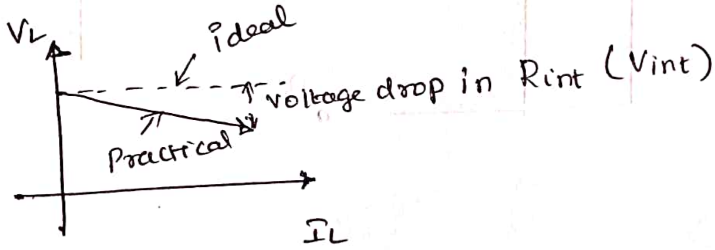
Ideal voltage source.





Practical voltage Source.



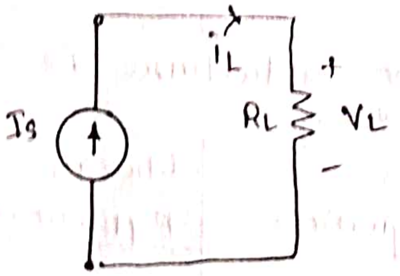


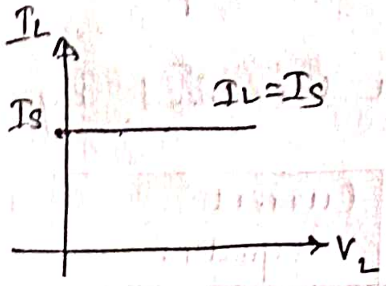
(ii) current source :-

(i) ideal

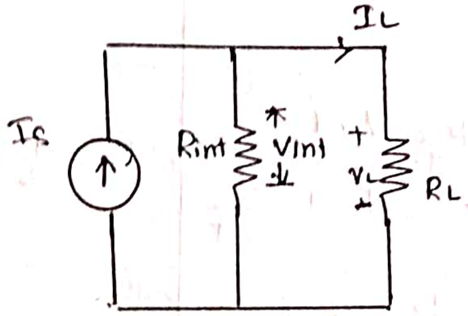
(ii) Practical.

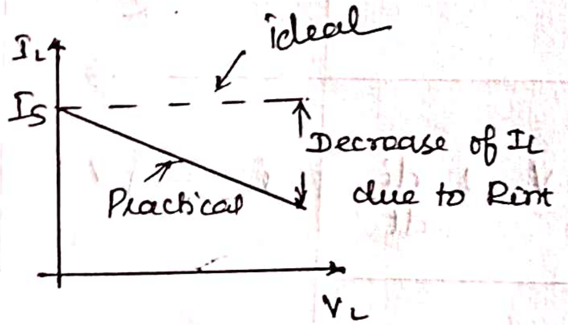
Ideal current source :-





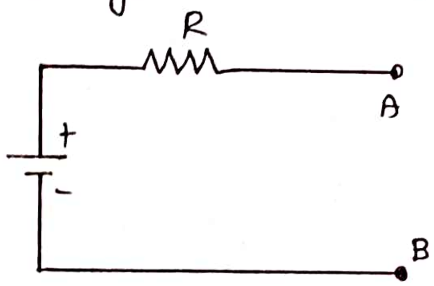
Practical current source :-

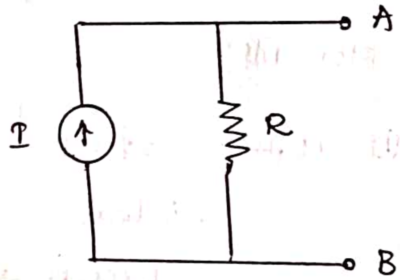




Source Transformation :-

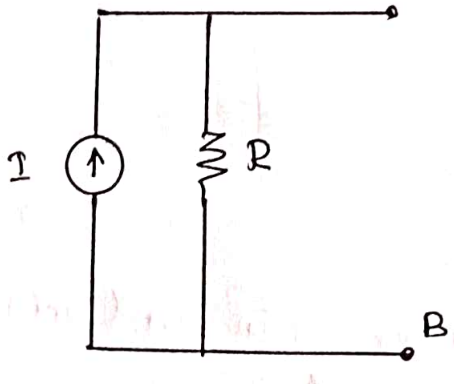
(i) Voltage source to current source

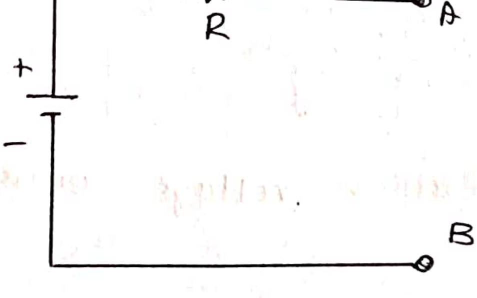




$$I = \frac{V}{R}$$

(ii) Current source to voltage source :-





Ohm's law:-

"At constant temperature, the current flowing through a conductor will be directly proportional to the applied voltage across the two ends of the conductor".

$$V \propto I$$

$$V = IR.$$

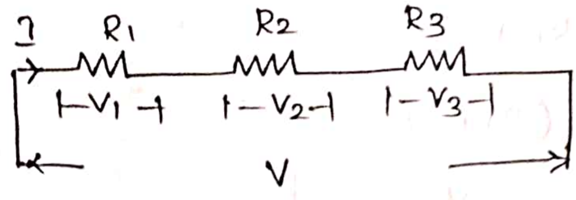
Limitations of ohm's law:-

(i) It is not applicable to non-Linear devices such as diode, voltage regulator etc.

(ii) It will not be well and good for non-metallic conductors.

(iii) It is not suitable for devices or places where the temperature is varying.

Series circuit:-



$$V = V_1 + V_2 + V_3$$

$$V_1 = IR_1 \quad , \quad V_2 = IR_2 \quad , \quad V_3 = IR_3$$

$$V = IR_1 + IR_2 + IR_3$$

$$= I(R_1 + R_2 + R_3)$$

$$\frac{V}{I} = R_1 + R_2 + R_3$$

$$R_{eq} = R_1 + R_2 + R_3$$

For 'n' resistance in series,

$$R_{eq} = R_1 + R_2 + R_3 \ldots R_n.$$

Characteristics of Series Circuit:-

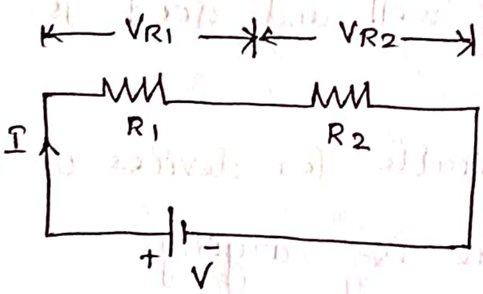
(i) Current through each resistance is same.

(ii) applied voltage (V) will be divided across each resistance.

$$V = V_1 + V_2 + V_3 \ldots V_n$$

(iii) Equivalent resistance equal to sum of individual resistance.

Voltage division in series circuit:-



$$V = V_{R1} + V_{R2}$$

$$V = IR_1 + IR_2$$

$$V = I(R_1 + R_2)$$

$$\therefore \boxed{I = \frac{V}{R_1 + R_2}}$$

Voltage drop across resistance (R_i) is $V_{R_1} = I \times R_1$

$$= \left[\frac{V}{R_1 + R_2}\right] \times R_1$$

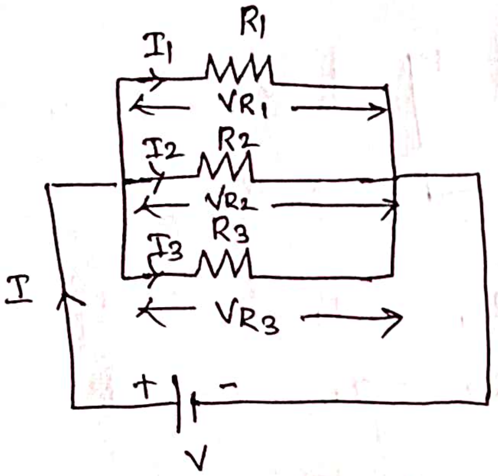
$$\therefore \boxed{V_{R1} = \frac{R_1}{R_1 + R_2} \times V}$$

voltage drop across the resistance (R_2) is $V_{R_2} = I \times R_2$

$$V_{R2} = \left[\frac{V}{R_1+R_2}\right] \times R_2$$

$$\therefore \boxed{V_{R2} = \frac{R_2}{R_1+R_2} \times V}$$

Parallel Circuit:



Apply ohm's law,

$$V_{R_1} = I_1 \times R_1 \quad V_{R2} = I_2 \times R_2 \ , \ V_{R3} = I_3 \times R_3$$

$$\therefore \quad V = V_{R1} \cancel{+} V_{R2} \cancel{+} V_{R3}$$

$$I_1 = \frac{V_1}{R_1} = \frac{V}{R_1}$$

Similarly $I_2 = \frac{V_2}{R_2} = \frac{V}{R_2}$, $I_3 = \frac{V_3}{R_3} = \frac{V}{R_3}$

$$I = I_1 + I_2 + I_3$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right].$$

$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

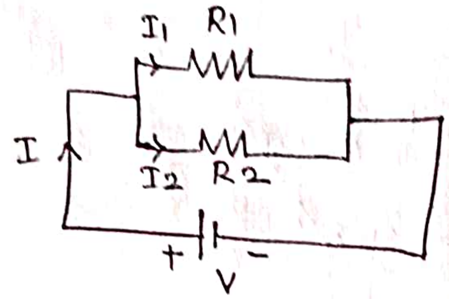
Characteristics of parallel circuit :-

(i) Voltage across all resistance is same.

(ii) Total current will be divided into number of paths

$$I = I_1 + I_2 + I_3 + \dots I_n$$

Current Division in parallel resistance circuit :-



Current through $R_1 = I_1$

Current through $R_2 = I_2$

Total current $I = I_1 + I_2$.

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}$$

$$\therefore V = I_1 R_1 = I_2 R_2$$

then

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$\therefore I_1 = I_2 \times \frac{R_2}{R_1}$$

$$\therefore I = I_2 \times \frac{R_2}{R_1} + I_2$$

$$I = I_2 \left[1 + \frac{R_2}{R_1} \right] = I_2 \left[\frac{R_1 + R_2}{R_1} \right]$$

$$\therefore I_2 = \left[\frac{R_1}{R_1 + R_2} \right] \times I$$

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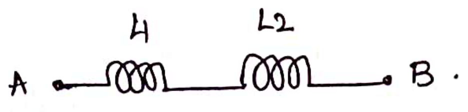
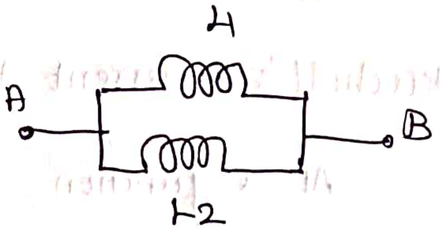
$$I_1 = \left[\frac{R_2}{R_1 + R_2} \right] \times I.$$

Inductors in series and parallel:-

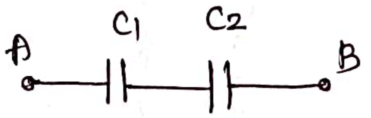


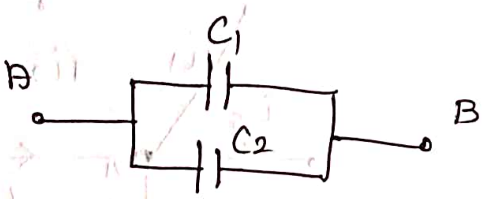


$$L_{eq} = L_1 + L_2$$

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

capacitors in series and parallel:-





$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{eq} = C_1 + C_2.$$

Open circuit and short circuit:-

when any two junction of a circuit are disconnected, those two junction are said to be open circuit. The resistance of a open circuited path is infinity. current in open circuit path is zero.

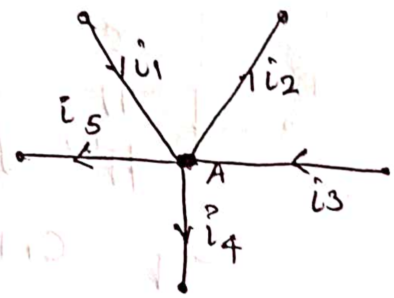
when any two junction of a circuit are connected directly to each other with a wire, they are said to be short circuited. The resistance of a short circuited path is zero.

Type of circuit	Resistance (R)	current (I)	Voltage (V)
short circuit	$R_{sc} = 0$	$I_{sc} = \infty$	$V_{sc} = 0$
open circuit	$R_{oc} = \infty$	$I_{oc} = 0$	$V_{oc} = \infty$

Kirchoff's Laws:-

(i) Kirchoff's Current Law:-

At a junction, total current flowing towards the junction is equal to total current flowing away from the junction.

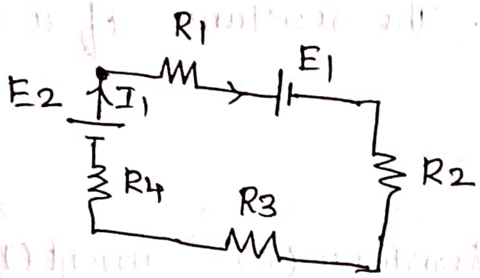


Current entering = current leaving.

$$i_1 + i_3 = i_2 + i_4 + i_5$$

(ii) Kirchoff's Voltage Law:-

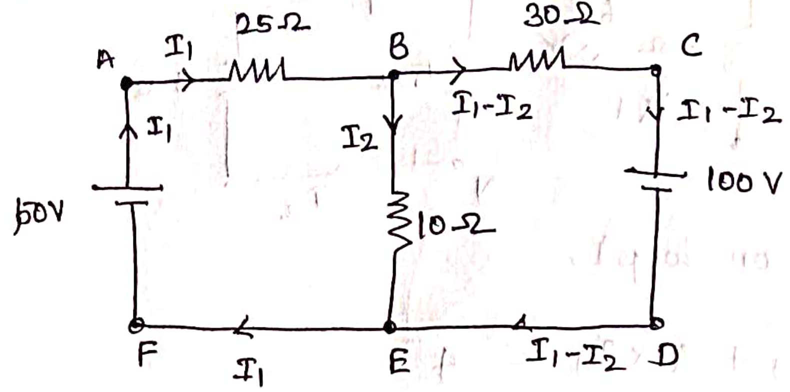
In any closed circuit or network the algebraic sum of product of current and resistance (voltage drop) across the circuit elements of any closed path is equal to the algebraic sum of the emf's in the path.



$$E_1 + E_2 = I_1 R_1 + I_1 R_2 + I_1 R_3 + I_1 \times R_4$$

potential rise = potential drop.

Apply Kirchoff's voltage law and current law and find the value of branch currents.



Loop ABEFA, Apply KVL

$$-25 I_1 - I_2 10 + 50 = 0$$

$$25 I_1 + 10 I_2 = 50$$

$\div 5$, $\quad 5 I_1 + 2 I_2 = 10 \quad \text{—— (1)}$

Loop BCDEB, apply KVL,

$$-30(I_1 - I_2) - 100 + 10 I_2 = 0.$$

$$-30 I_1 + 30 I_2 + 10 I_2 - 100 = 0.$$

$$-30 I_1 + 40 I_2 - 100 = 0.$$

$$30 I_1 - 40 I_2 = 100$$

$\div 10$ $\quad 3 I_1 - 4 I_2 = 10 \quad \text{—— (2)}$

Equate eqn (1), & (2), we get

eqn (1) × 2 $\Rightarrow$ $\quad 10 I_1 + 4 I_2 = 20$

$$3 I_1 - 4 I_2 = 10$$

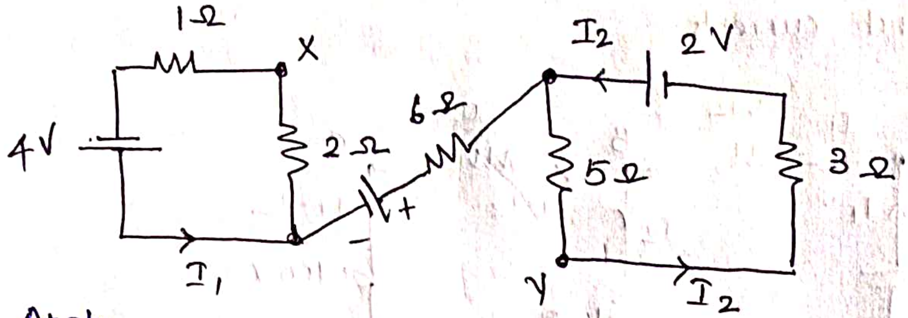
$$13 I_1 = 30$$

$$\boxed{I_1 = 0.772\ A}$$

apply in eqn (1), $\quad 5 \times 0.772 + 2 I_2 = 10.$

$$\boxed{I_2 = 3.07\ A}$$

obtain the potential difference V_{xy} in the circuit shown



Apply KVL on loop 1,

$$1 \times I_1 + 2 \times I_1 = 4$$

$$3 I_1 = 4$$

$$I_1 = 4/3$$

$$\boxed{I_1 = 1.33\ A}$$

Apply KVL on loop 2,

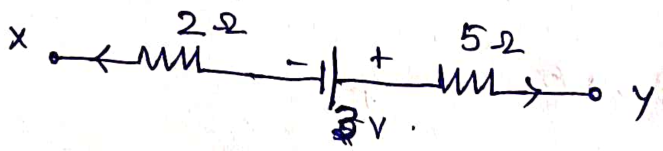
$$5 I_2 + 3 I_2 = 2$$

$$8 I_2 = 2$$

$$I_2 = 2/8$$

$$\boxed{I_2 = 0.25\ A}$$

Since no current flow through 6Ω resistance. It is not included.

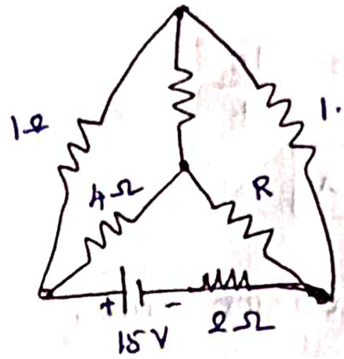


$$V_{xy} = 2 I_1 + 3 - 5 I_2$$

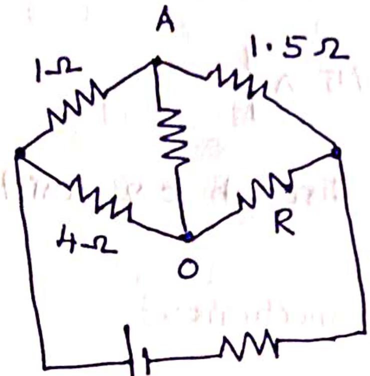
$$= 2 \times 1.33 + 3 - 5 \times 0.25$$

$$\boxed{V_{xy} = 4.41\ V}$$

Find the value of R and current flowing through it in the circuit when the current in the branch OA is zero.



The given circuit is redrawn as



As the current through OA is zero, the circuit is in balanced condition.

The product of resistances of opposite arms must be equal in balanced conditions.

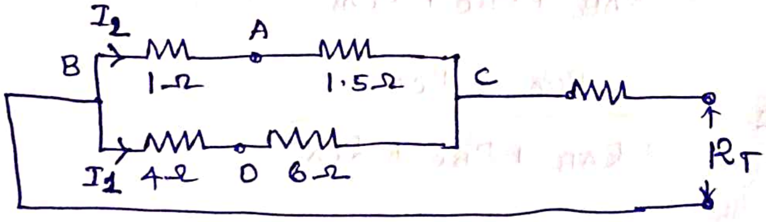
ie current in the branch OA is zero.

$$\frac{R}{1.5} = \frac{4}{1}$$

$$R = 4 \times 1.5$$

$$\boxed{R = 6\Omega}$$

The current through OA is zero, it can be removed.



$$\text{Total resistance} = \frac{2.5 \times 10}{2.5 + 10} + 2$$

$$\boxed{R_T = 4\,\Omega}$$

$$I = \frac{V}{R_T} = \frac{15}{4}$$

$$\boxed{I = 3.75\text{ A}}$$

Apply current division rule,

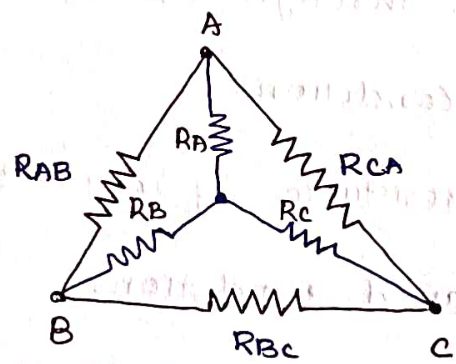
$$I_1 = \frac{3.75 \times 2.5}{10 + 2.5}$$

$$\boxed{I_1 = 0.75\text{ A}}$$

Current flowing through 6Ω resistor $= 0.75$ A.

Star and Delta Connections :-

(i) Delta to star transformation :-



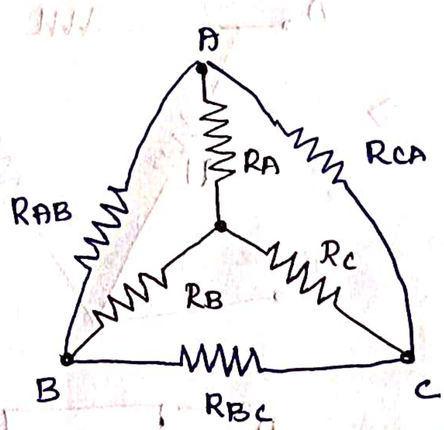
star resistances are R_A, R_B & R_C

$$R_A = \frac{R_{AB} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_B = \frac{R_{BC} \times R_{AB}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{BC} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

Star to Delta Transformation



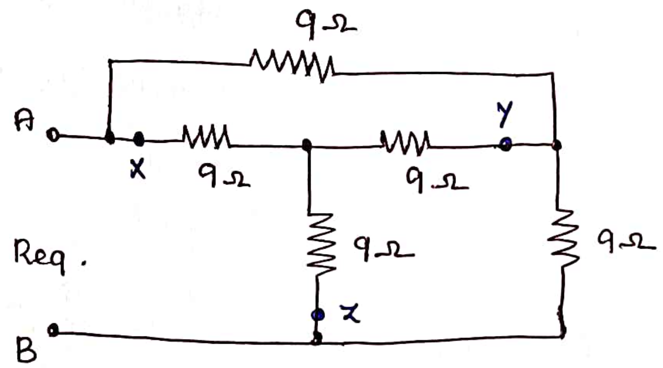
Delta resistances are R_{AB}, R_{BC} & R_{CA}.

$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

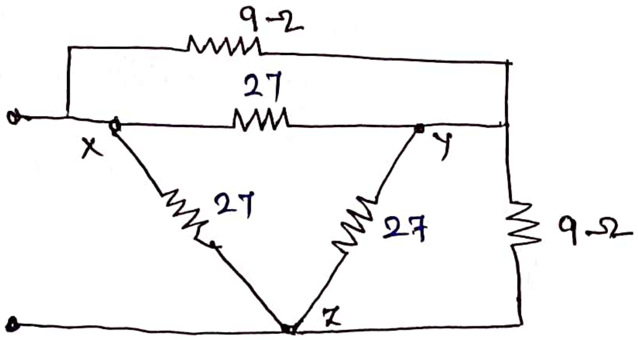
$$R_{BC} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$R_{CA} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}.$$

Determine the equivalent resistance between A & B.



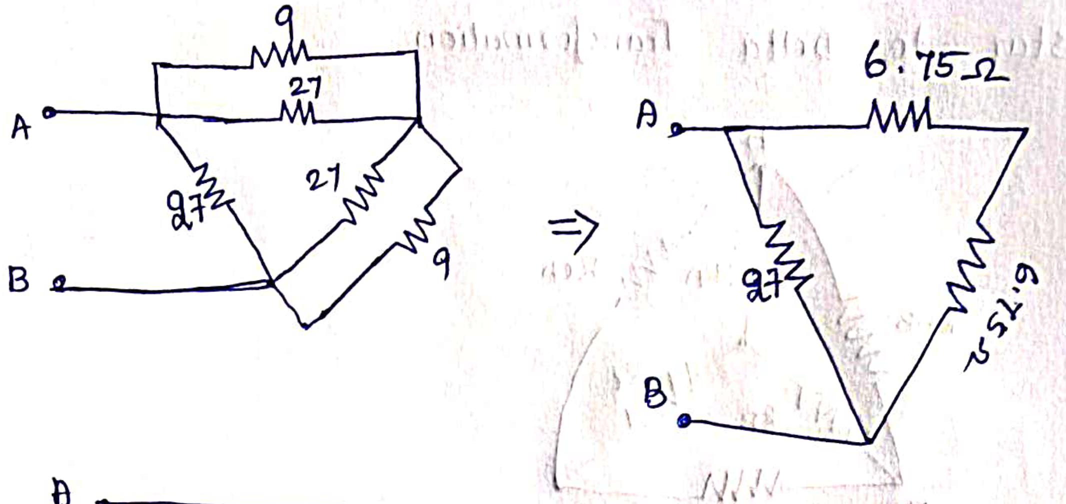
x y z terminal → convert star to delta.

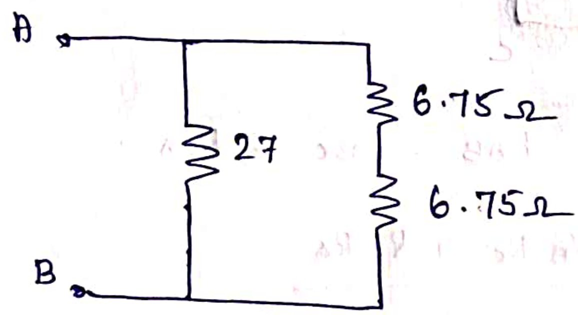


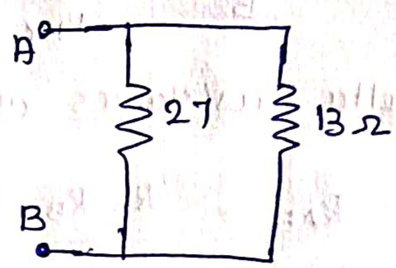
$R_{xy} = 3 \times 9 = 27$

$R_{yz} = 27$

$R_{zx} = 27$







$$R_{AB} = \frac{27 \times 13.5}{27 + 13.5}$$

$$\boxed{R_{AB} = 9\ \Omega}$$

Basic Circuit Analysis - AC Circuits

Alternating current:-

Alternating current (AC) is the current which changes its magnitude and direction at regular interval of time.

Advantages of AC supply:-

(i) It is possible to set up (or) step down the voltage to the required value, by the transformer.

(ii) when a high voltage is transmitted, in the transmission line, the current will be minimum. Hence the I^2R loss in the conductor will be minimum and therefore transmission system will be efficient and economical.

(iii) AC motors are simple and robust in construction, cheaper and requires less maintenance.

Basic Definitions in AC circuits.

Instantaneous voltage $V = V_m \sin\omega t$

Instantaneous current $I = I_m \sin\omega t$

Amplitude or Peak or Maximum Value:-

The maximum value attained by the alternating quantity during positive and negative half cycle is called as amplitude or peak value.

Frequency:-

The total number of cycles made by an alternating quantity per second is known as frequency.

Relation between frequency and time period.

$$f = \frac{1}{T}$$

Relation between frequency and angular velocity

$$V = V_m \sin \omega t.$$

$$\omega = 2\pi f.$$

Average value or Mean value of alternating quantity:-

The value which is obtained by averaging the instantaneous values of the alternating quantity over one complete cycle is the average value of the quantity

$$I_{avg} = \frac{2 I_m}{\pi} = \frac{\text{Area under the half cycle}}{\text{Base time for one cycle.}}$$

$$\boxed{I_{avg} = 0.637\ I_m}$$

$$\boxed{V_{avg} = 0.637\ V_m}$$

Root Mean square (RMS) - Effective value of a alternating quantity:-

$$I_{rms} = \sqrt{\frac{\text{Area under the squared curve}}{\text{Base}}}$$

$$I_{rms} = I_{max}/\sqrt{2}$$

$$\boxed{I_{rms} = 0.707\ I_{max}}$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$\boxed{V_{rms} = 0.707\ V_{max}}$$

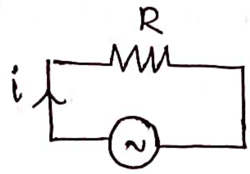
Form factor and crest factor :-

$$\text{Form factor } k_f = \frac{\text{Rms Value}}{\text{Average value}}$$

$$\text{crest factor (Peak factor) } k_a = \frac{\text{Maximum Value}}{\text{RMS value}}.$$

Simple AC circuits :-

(i) AC through pure resistance :-



$V = V_m \sin\omega t$

using ohm's law,

$$i = \frac{V}{R} = \frac{V_m \sin\omega t}{R}$$

$$i = \frac{V_m}{R} \sin\omega t$$

$$i = I_m \sin\omega t \qquad \because I_m = \frac{V_m}{R}$$

Power :-

Instantaneous power $P = V \times i$

$$P = V_m \sin\omega t \times I_m \sin\omega t$$

$$P = V_m \times I_m \sin^2\omega t$$

$$P = V_m I_m \left[\frac{1-\cos 2\omega t}{2}\right]$$

$$P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t.$$

Average power, $P_{avg} = \frac{V_m I_m}{2}$

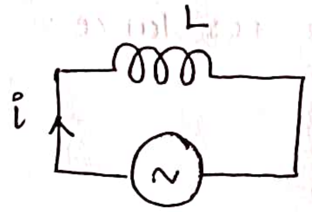
since P_{avg} of cosine wave is <u>zero</u>.

$$\therefore \quad P_{avg} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = V_{rms} \times I_{rms}.$$

also $\boxed{P_{avg} = VI}$

AC through pure inductor



$V = V_m \sin\omega t$

$$e = -L \times \frac{di}{dt} \quad -(1)$$

$$e = -V \quad -(2) \quad [\because \text{ lenz law}]$$

$$-V = -L \times \frac{di}{dt}$$

$$V = L \times \frac{di}{dt}$$

$$di = \frac{V}{L}\,dt.$$

integrating, $\quad i = \frac{1}{L}\int V_m \sin\omega t \,.\, dt$

$$i = \frac{V_m}{L}\left[-\frac{\cos\omega t}{\omega}\right]$$

$$i = \frac{-V_m}{\omega L} \cos \omega t$$ $\because \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

$$i = \frac{-V_m}{\omega L} \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2)$$

The instantaneous value of the current is given by

$$i = I_m \sin(\omega t - \varphi)$$

$$I_m = \frac{V_m}{\omega L}$$

$$I_m = \frac{V_m}{X_L}$$

where $X_L = \omega L = 2\pi f L$

Power :-

$$P = V \times i$$

$$= V_m \sin \omega t \times I_m \sin(\omega t - \pi/2)$$

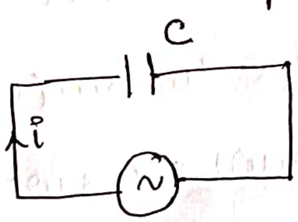
$$= V_m I_m \sin \omega t \sin(\omega t - \pi/2)$$

$$P = V_m I_m \sin \omega t (-\cos \omega t)$$

$$P = -\frac{V_m I_m}{2} \sin(2\omega t)$$ $\because \sin \omega t - \pi/2 = -\cos \omega t$

$$P_{avg} = 0.$$

AC through pure capacitor:-



$V = V_m \sin \omega t$

$$c = \frac{q}{V}$$

$$q = CV$$

$$i = \frac{dq}{dt}$$

$$i = \frac{d}{dt}(CV)$$

$$i = C \times \frac{dv}{dt}$$

$$= C \times \frac{d}{dt}(V_m \sin \omega t)$$

$$i = C V_m (\cos \omega t)\, \omega$$

$$i = V_m (\omega C) \cos \omega t$$

$$i = \frac{V_m}{1/\omega C} \times \sin(\omega t + \pi/2)$$

$$i = \frac{V_m}{X_c} \sin(\omega t + \pi/2)$$

The instantaneous value of current is given by

$$i = I_m \sin(\omega t + \varphi)$$

$$I_m = \frac{V_m}{1/\omega C} = \frac{V_m}{X_c}$$

$$X_c = \frac{1}{2\pi f C}$$

Power:-

Instantaneous power $P = V \times i$

$$P = V_m \sin \omega t \times I_m \sin(\omega t + \pi/2)$$

$$P = V_m I_m \sin \omega t \times \cos \omega t$$

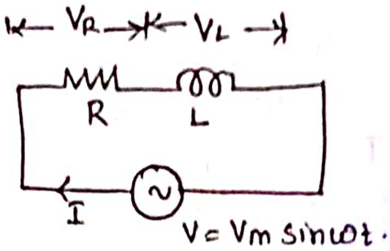
$$P = V_m I_m \sin\omega t \cos\omega t$$

$$= V_m I_m \frac{\sin 2\omega t}{2}$$

$$P = \frac{V_m I_m}{2} \sin 2\omega t$$

$$P_{avg} = 0 .$$

Ac through series R-L circuit



$$V = V_R + V_L$$

$$V_R = I \times R$$

$$V_L = I \times X_L$$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{I^2 R^2 + I^2 X_L^2}$$

$$= I \sqrt{R^2 + X_L^2}$$

$$V = I \times Z .$$

$$Z = \sqrt{R^2 + X_L^2} \quad \Omega$$

phasor Form :-

$$Z = R + j X_L \quad \Omega$$

$$Z = |Z| \angle \varphi$$

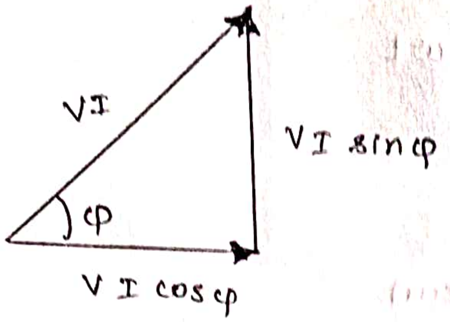
$$|Z| = \sqrt{R^2 + X_L^2} \qquad \varphi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$R = Z \cos\varphi$$

$$X_L = Z \sin\varphi$$

Power and Power triangle

$$P_{avg} = V I \cos\varphi$$



Apparent power :-

$$S = V I$$

Real power :-

$$P = V I \cos\varphi$$

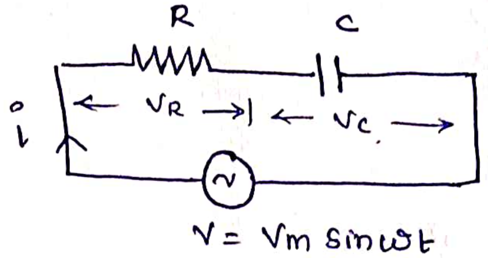
Reactive power :-

$$Q = VI \sin\varphi$$

Power factor :-

$$\cos\varphi = \frac{R}{|Z|}$$

AC through series RC circuit :-



$$V = V_R + V_c$$

$$V_R = I \times R$$

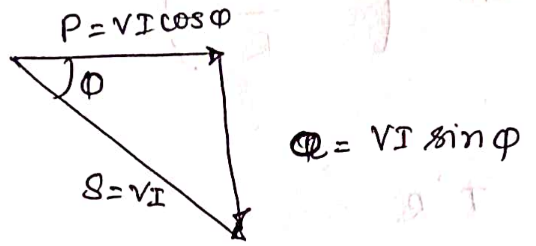
$$V_c = I \times X_c$$

$$Z = \sqrt{R^2 + X_c^2}$$

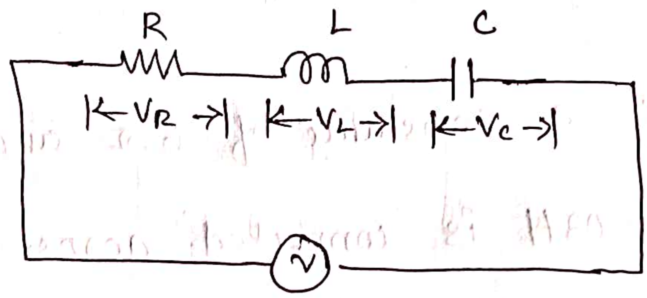
$$Z = |Z| \angle -\varphi \quad , \quad \varphi = \tan^{-1}\left(\frac{-X_c}{R}\right)$$

Power and Power Triangle :-

$$P_{avg} = VI\cos\phi$$



AC Through R-L-C series circuit :-



$$V = V_m \sin \omega t$$

$$V = V_R + V_L + V_C$$

$$V = IR + IX_L + IX_C$$

case 1 :- $X_L > X_C$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

case 2 : $X_L < X_C$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

case 3 : $X_L = X_C$

$$Z = R$$

Impedence :-

$$Z = R + jX_L - jX_C$$

$$Z = R + j(X_L - X_C)$$

$$Z = R + jX \qquad \because X = X_L - X_C$$

$$\cos\varphi = \frac{R}{Z}$$

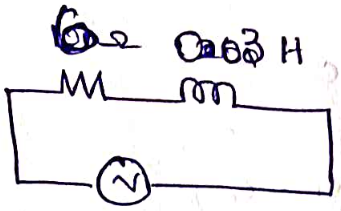
$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Power :-

$$P_{avg} = I^2 R$$

$$P = V I \cos\varphi$$

A coil having a resistance of 6 Ω and an inductor of 0.03 H is connected across a 100V, 50 Hz supply. Calculate (i) current (ii) power angle between the current and voltage. (iii) power factor (iv) power.



$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.03$$

$$\boxed{X_L = 9.42\ \Omega}$$

$$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{6^2 + 9.42^2}$$

$$\boxed{Z = 11.16\ \Omega}$$

(i) current :-

$$I = \frac{V}{Z} = \frac{100}{11.16}$$

$$\boxed{I = 8.96\ A}$$

(ii) phase angle :-

$$\cos\varphi = \frac{R}{Z}$$

$$\varphi = \cos^{-1}\left(\frac{R}{Z}\right)$$

$$\varphi = \cos^{-1}(0.537)$$

$$\boxed{\varphi = 57^\circ 31'}$$

(iii) power factor :-

$$\cos\varphi = \frac{R}{Z}$$

$$= \frac{6}{11.6}$$

$$\boxed{\cos\varphi = 0.537\ (\text{lag})}$$

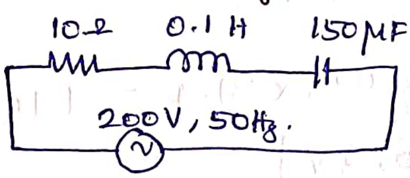
(iv) Power :-

$$P = VI\cos\varphi$$

$$= 100 \times 8.96 \times 0.537$$

$$\boxed{\text{Power } P = 481.152\ W}$$

A coil of resistance 10Ω and an inductance of 0.1 H is connected in series with a condenser (or) capacitance of 150 μF across a 200 V, 50 Hz supply. Calculate (i) inductive reactance (ii) capacitive reactance (iii) net reactance (iv) current (v) power factor (vi) voltage across R, L and C.



(i) Inductive Reactance :-

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.1$$

$$\boxed{X_L = 31.4\ \Omega}$$

(ii) Capacitive Reactance :-

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$$

$$\boxed{X_C = 21.23\ \Omega}$$

(iii) Net reactance :-

$$X = X_L - X_C = 31.4 - 21.23$$

$$\boxed{X = 10.17\ \Omega}$$

(iv) Impedence :-

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{10^2 + 10.17^2}$$

$$\boxed{Z = 14.26\ \Omega}$$

(v) current :-

$$I = \frac{V}{Z}$$

$$= \frac{200}{14.26}$$

$$\boxed{I = 14.025\ A}$$

(vi) power factor :-

$$\cos\varphi = \frac{R}{Z} = \frac{10}{14.26}$$

$$\boxed{\cos\varphi = 0.701 \text{ lag}}$$

(vii) voltage across each element :-

* $V_R = IR = 14.025 \times 10 = 140.25\ V$

$$\boxed{V_R = 140.25\ V}$$

* $V_L = I \times X_L = 14.025 \times 31.4 = 440.3\ V$

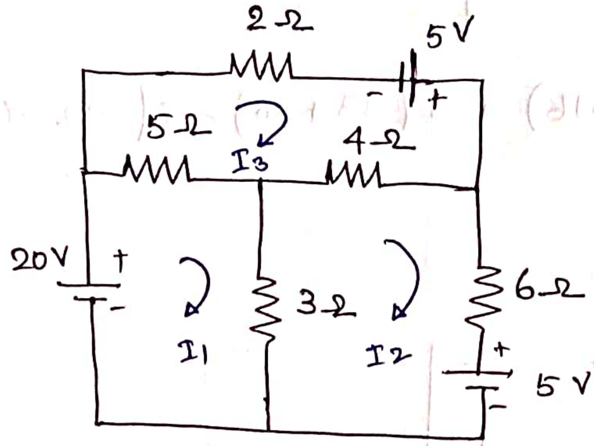
$$\boxed{V_L = 440.3\ V}$$

* $V_C = I \times X_C = 14.025 \times 21.23 = 297.75\ V$

$$\boxed{V_C = 297.75\ V}$$

Mesh Analysis:-

using mesh analysis, find the current through 4Ω.



Apply KVL to 3 loops,

Loop 1, $-5(I_1-I_3)-3(I_1-I_2)+20=0.$

$$-5I_1+5I_3-3I_1+3I_2=-20.$$

$$-8I_1+3I_2+5I_3=-20.$$

$$8I_1-3I_2-5I_3=20 \quad \text{——} \quad (1)$$

Loop 2, $-4(I_2-I_3)-6(I_2)-3(I_2-I_1)-5=0.$

$$-4I_2+4I_3-6I_2-3I_2+3I_1-5=0$$

$$3I_1-13I_2+4I_3=5 \quad \text{——} \quad (2)$$

Loop 3, $-2I_3+5-4I_3+4I_2-5I_3+5I_1=0.$

$$5I_1+4I_2-11I_3=-5$$

$$-5I_1-4I_2+11I_3=5 \quad \text{——} \quad (3).$$

$$\begin{bmatrix} 8 & -3 & -5 \\ 3 & -13 & 4 \\ -5 & -4 & 11 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 5 \\ 5 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -3 & -5 \\ 3 & -13 & 4 \\ -5 & -4 & 11 \end{vmatrix}$$

$$= 8(-143+16) + 3(33+20) - 5(-12-65)$$

$$\boxed{\Delta = -472}$$

$$\Delta_2 = \begin{vmatrix} 8 & 20 & -5 \\ 3 & 5 & 4 \\ -5 & 5 & 11 \end{vmatrix}$$

$$= 8(55-20) - 20(33+20) - 5(15+25) = -980$$

$$\boxed{\Delta_2 = -980}$$

$$\Delta_3 = \begin{vmatrix} 8 & -3 & 20 \\ 3 & -13 & 5 \\ -5 & -4 & 5 \end{vmatrix}$$

$$= 8(-65+20) + 3(15+25) + 20(-12-65)$$

$$\boxed{\Delta_3 = -1780}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{+980}{+472}$$

$$\boxed{I_2 = 2.076\ A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{+1780}{+472}$$

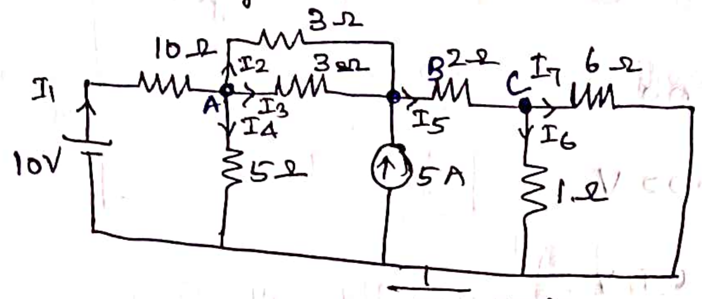
$$\boxed{I_3 = 3.7711\ A}$$

$$I_{4\Omega} = I_3 - I_2 = 3.7711 - 2.076$$

current through 4Ω resistor = 1.6949 A.

Nodal Analysis:-

Determine the voltage at each Node for the given circuit.



reference node $V_d = 0$

Apply KCL at node A,

$$I_1 - I_2 - I_3 - I_4 = 0.$$

$$\left(\frac{10 - V_A}{10}\right) - \left(\frac{V_A - V_B}{3}\right) - \left(\frac{V_A - V_B}{3}\right) - \left(\frac{V_A - V_D}{5}\right) = 0.$$

$$\left[\frac{1}{10} + \frac{1}{5} + \frac{1}{3} + \frac{1}{3}\right] V_A - \left(\frac{1}{3} + \frac{1}{3}\right) V_B = 1$$

$$0.967 V_A - 0.667 V_B = 1 \quad \text{——— (1)}$$

Apply KCL at node B,

$$I_2 + I_3 + 5 - I_5 = 0$$

$$\frac{V_A - V_B}{3} + \frac{V_A - V_B}{3} + 5 - \frac{V_B - V_C}{2} = 0$$

$$-\left[\frac{1}{3} + \frac{1}{3}\right] V_A + \left[\frac{1}{3} + \frac{1}{3} + \frac{1}{2}\right] V_B - \left[\frac{1}{2}\right] V_C = 5$$

$$-0.667 V_A + 1.167 V_B - 0.5 V_C = 5 \quad \text{——— (2)}$$

Apply KCL at node C,

$$I_5 - I_6 - I_7 = 0.$$

$$\left(\frac{V_B - V_C}{2}\right) - \left(\frac{V_C - V_D}{1}\right) - \left(\frac{V_C - V_D}{6}\right) = 0.$$

$$-\frac{1}{2} V_B + \left(\frac{1}{2} + \frac{1}{1} + \frac{1}{6}\right) V_C = 0.$$

$$-0.5 V_B + 1.667 V_C = 0 \quad \text{——— (3)}$$

Equate eqn 1, 2 & 3, we get

$V_A = 8.08\ V$

$V_B = 10.22\ V$

$V_C = 3.07\ V.$

unit - II

Network Reduction and Theorems

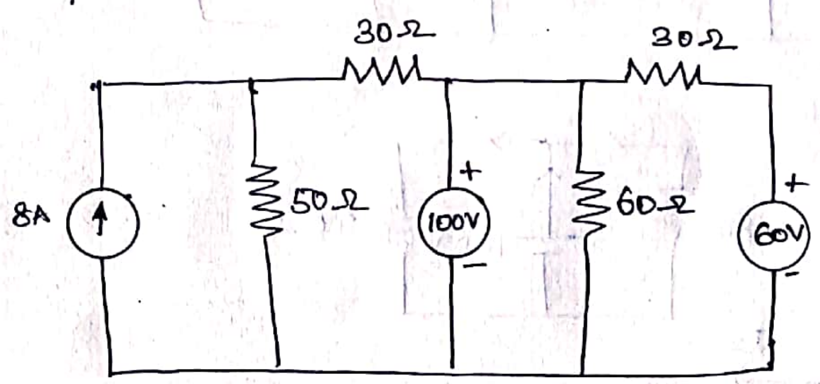
Super position Theorem :-

In any linear multi source complex circuit, the voltage or current in a particular resistor will be equal to the algebraic sum of the voltage or current in that resistor, with the consideration of one source at a time and all other sources are turned off, or made inoperative.

Limitations of super position Theorem :-

* It involves more work in the network solution
* It based on linearity principle. It is not applicable to power due to each sources, individually.
* only applicable for linear circuit.

Determine I_1 current in the circuit shown. using super position theorem.

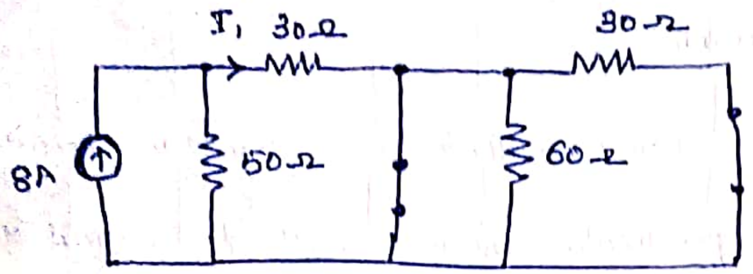


Solution :-

current source → open circuit

voltage source → short circuit.

step 1 : Consider current source . (8 A) . Remove all volta

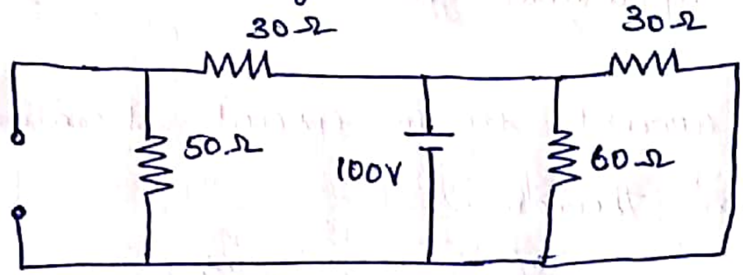


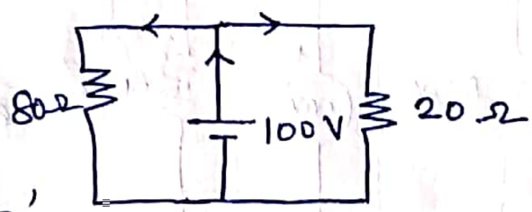
The current is flowing through short circuit path, there will be no through the parallel combination of 30 Ω and 60 Ω resistance. Now apply current devision rule.

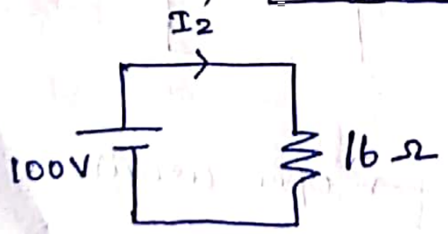
$$I_1' = I_{30\Omega} = 8 \times \frac{50}{30+50}$$

$$\boxed{I_1' = 5\,A}$$

Step 2 : Considering voltage source (100 V) . Remove other voltage & current source.

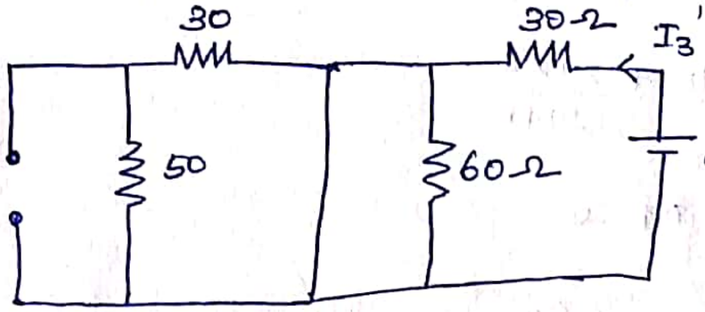




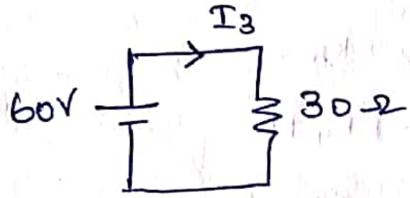


$$I_2' = \frac{100}{16} \quad , \quad \boxed{I_2' = 6.25\,A}$$

Step 3 :- consider 60 V source only.



I_3' flowing through short circuit path. so that 50 Ω & 30 Ω are eliminated. and also 60 Ω.



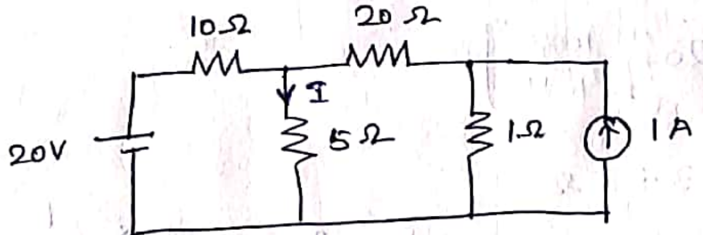
$I_3' = \frac{60}{30}$, $\boxed{I_3' = 2A}$

Step 4 :- Sum of all currents.

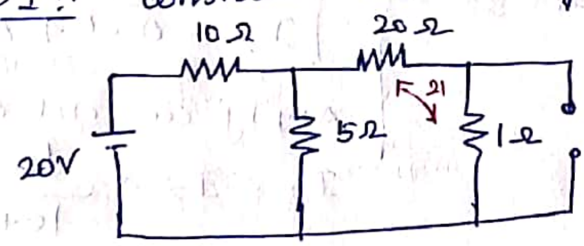
$I_1 = I_1' + I_2' + I_3' = -5 + 6.25 - 2$

$\boxed{I_1 = -0.75 \text{ A}}$

Solve for current through 5 Ω resistance by principle of super position theorem.



Step 1 :- consider 20 V only.



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$$R_{eq1} = \frac{21 \times 5}{21+5}$$

$$R_{eq2} = 10 + \frac{21\times5}{21+5}$$

$$R_{eq2} = 14.04\ \Omega$$

$$I_1^{\bullet} = \frac{V_1}{R_{eq2}} = \frac{20}{14.04}$$

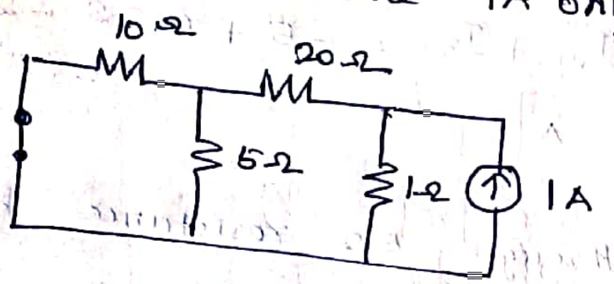
$$\boxed{I_1^{\bullet} = 1.425\ A}$$

$$I_1' = I_1 \times \frac{R_2+R_4}{R_2+R_4+R_3}$$

$$= 1.425 \times \frac{20+1}{20+1+5}$$

$$\boxed{I_1' = 1.15\ A}$$

Step 2 : current source 1A only



$$R_{eq1} = \frac{10\times5}{10+5}$$

$$R_{eq2} = 20 + \left[\frac{10\times5}{10+5}\right]$$

$$R_{eq2} = 23.33\ \Omega$$

$$R_{eq3} = \frac{1\times 23.33}{1+23.33}$$

$$R_{eq3} =$$

$$I_2 = 1 \times \frac{1}{23.3+1}$$

$$\boxed{I_2 = 0.04\ A}$$

Apply current division rule

$$I_3 = I_2 \times \frac{10}{10+5}$$

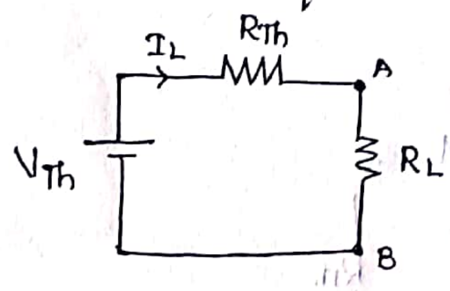
$$I_3 = 0.0267$$

$I_2' = I_2 + I_3 = 1.15 + 0.0267$

$\boxed{I_2' = 1.1767\ A}$

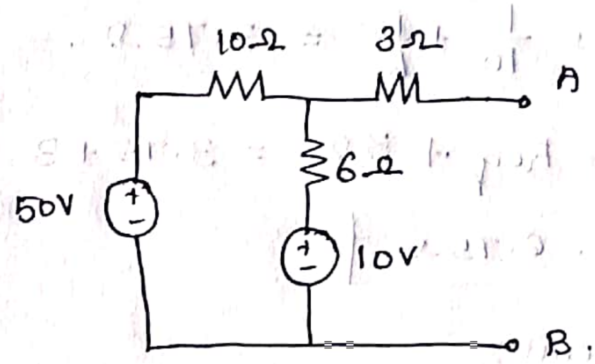
Thevenin's Theorem:-

Thevenin's theorem states that, any linear circuit with two terminals, can be replaced by an equivalent circuit consisting of a voltage source (V_{Th}) in series with an equivalent resistance (R_{Th}).



$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Find the thevenin's equivalent circuit for the circuit shown.



Step 1 : To find V_{Th}.

using mesh analysis,

$$-10\,I_1 - 6\,I_1 - 10 + 50 = 0.$$

$$16\,I_1 = 40$$

$$I_1 = 40/16$$

$$\boxed{I_1 = 2.5\ A}$$

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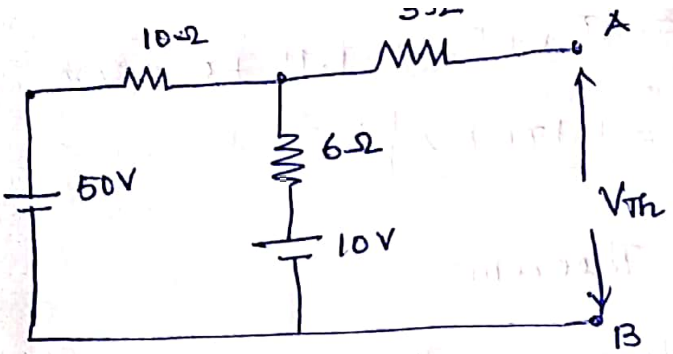
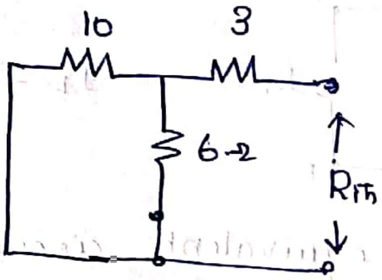


$$V_{oc} = V_{Th} = (3\times 0) + (6\times 2.5) + 10$$

$$= 0 + 15 + 10$$

$$\boxed{V_{Th} = 25\ V}$$

To find R_{Th}:-

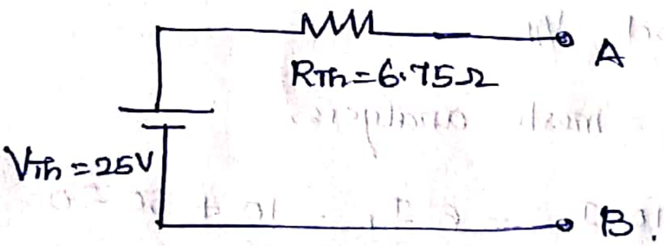


$$R_{eq} = \frac{1}{10} + \frac{1}{6} = 3.75\,\Omega.$$

$$R_{Th} = R_{eq} + 3\Omega = 3.75 + 3$$

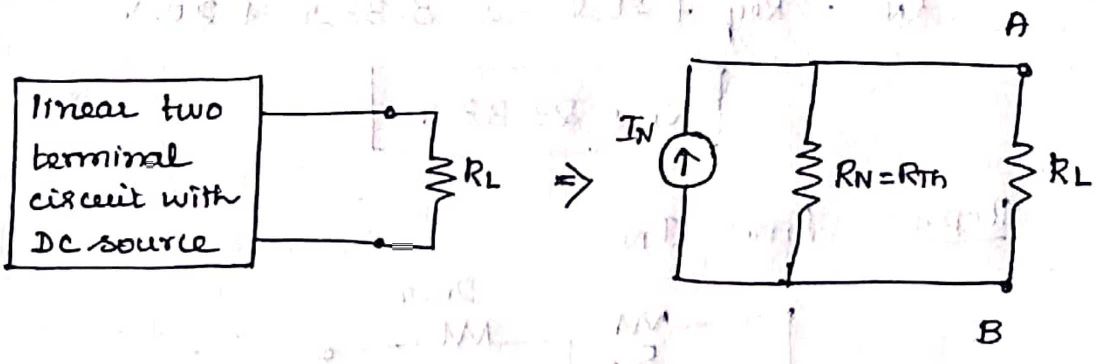
$$\boxed{R_{Th} = 6.75\,\Omega}$$

Equivalent circuit:-

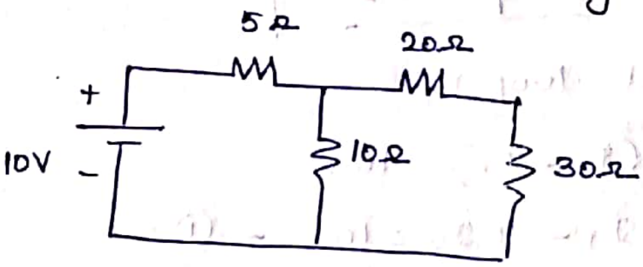


Norton's Theorem:-

Theorem states that, any circuit with two terminals, can be replaced by an equivalent circuit consisting a current source (I_N) in parallel with a equivalent Resistance.



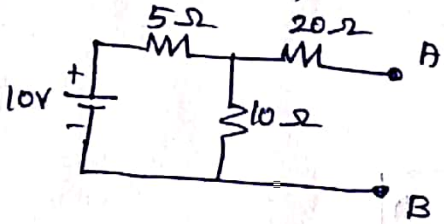
Pbm:- Find Norton's equivalent model for the circuit, find the current through 30Ω resistance using Norton's theorem.



Step 1: Find Load resistance.

$R_L = 30\Omega$

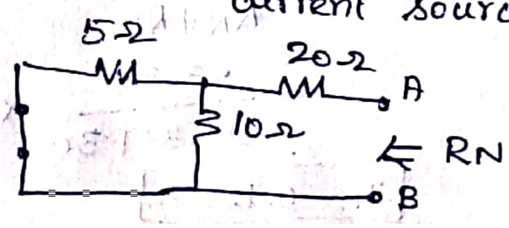
Step 2: Remove R_L,



Step 3:- Find Norton's Resistance.

Remove : Voltage souce → short circuited

current source → open circuit).



$5\,\Omega$ and $10\,\Omega$ are connected in parallel.

$$R_{eq} = \frac{5 \times 10}{5+10}$$

$$R_{eq} = 3.33\,\Omega.$$

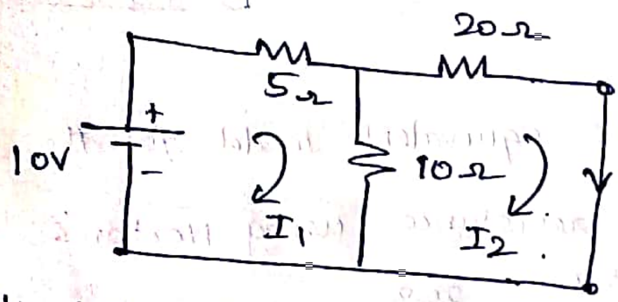
Now

$3.33\,\Omega$ and $20\,\Omega$ are connected in series.

$$R_N = R_{eq} + 20\,\Omega = 3.33\,\Omega + 20\,\Omega.$$

$$\boxed{R_N = 23.33\,\Omega}$$

Step 4: Find I_N.



Apply KVL at loop 1,

$$5 I_1 + 10(I_1 - I_2) = 10\,V$$

$$15 I_1 - 10 I_2 = 10 \quad \text{--- (1)}.$$

Apply KVL at loop 2,

$$20 I_2 + 10(I_2 - I_1) = 0.$$

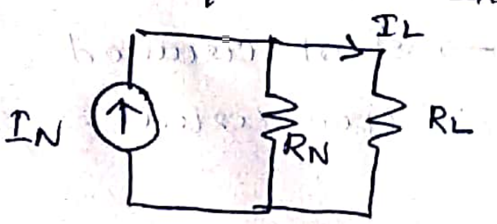
$$-10 I_1 + 30 I_2 = 0 \quad \text{--- (2)}.$$

$$I_1 = 0.857\,A$$

$$I_2 = 0.285\,A$$

$$\therefore \boxed{I_N = I_2 = 0.285\,A}$$

Step 5: Equivalent circuit.



$$I_L = \frac{R_N}{R_N + R_L} \times I_N$$

$$= \frac{23.33}{23.33 + 30} \times 0.285$$

$$\boxed{I_L = 0.125\,A}$$

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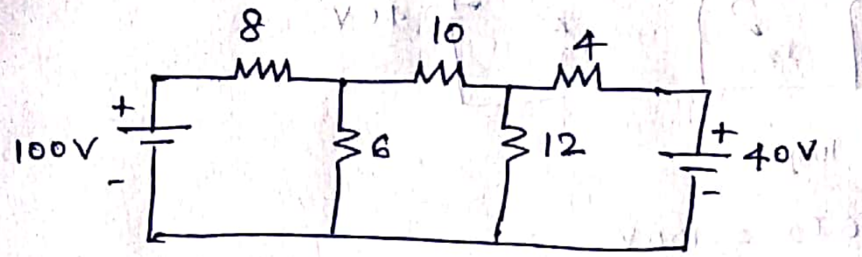
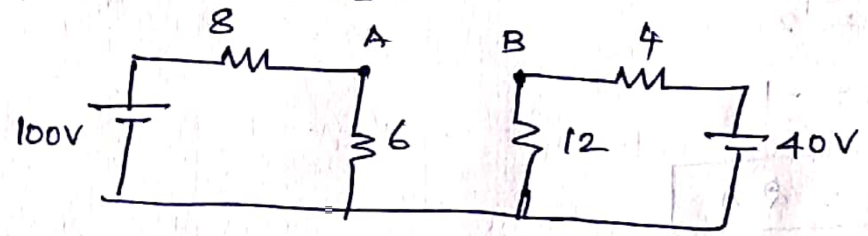
For the circuit shown below, find the Norton's equivalent across 10Ω resistance. Also find the current through 10Ω resistance :



Step 1 : Find R_L.

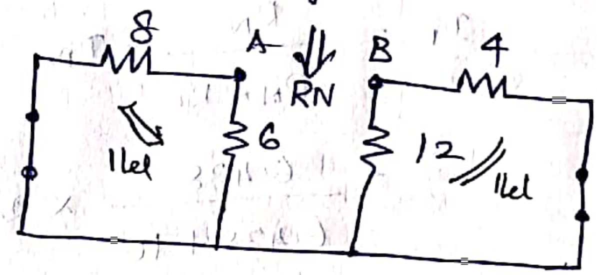
$$\boxed{R_L = 10\Omega}$$

Step 2 : Remove R_L :



Step 3 : Find R_N.

Remove voltage source → short circuited.



8Ω and 6Ω are connected in parallel,

$$Req_1 = \frac{8 \times 6}{8+6} = 3.428\ \Omega$$

4Ω and 12Ω are connected in parallel.

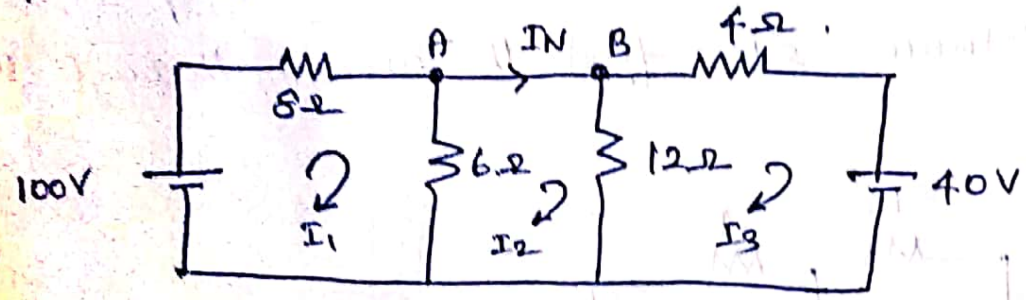
$$Req_2 = \frac{4 \times 12}{4+12} = 3\ \Omega$$

Now Req_1 & Req_2 are connected in series.

$$\therefore R_N = Req_1 + Req_2 = 3.428 + 3\ \Omega$$

$$\boxed{R_N = 6.428\ \Omega}$$

Step 4 : Find Norton's " current I_N.



Apply KVL on loop 1,

$$14 I_1 - 6 I_2 = 100V$$

$$-6 I_1 + 18 I_2 - 12 I_3 = 0$$

$$-12 I_2 + 16 I_3 = -40$$

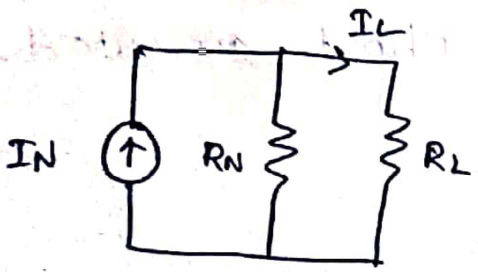
$$I_1 = 8$$

$$I_2 = 2$$

$$I_3 = -1$$

$$\boxed{I_N = I_2 = 2\ A}$$

Step 5 : Equivalent circuit :



$$I_L = \frac{R_N}{R_N + R_L} \times I_N$$

$$= \frac{6.428}{6.428 + 10} \times 2$$

$$\boxed{I_L = 0.782\ A}$$

--- x --- x ---

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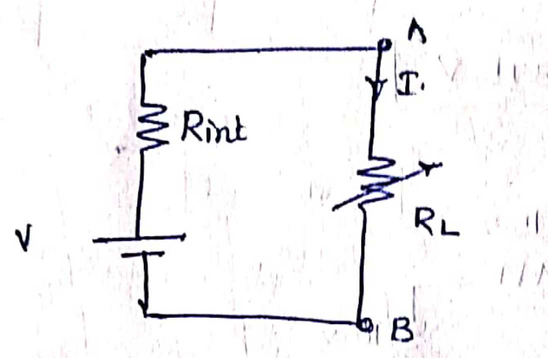
Maximum Power Transfer Theorem:-

statement :-

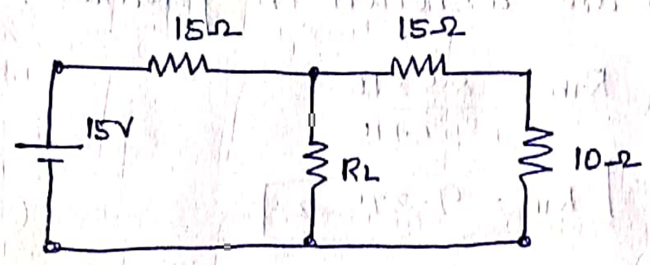
Theorem states that, Maximum power will be delivered from a voltage source to a load, when the load resistance is equal to the source resistance.



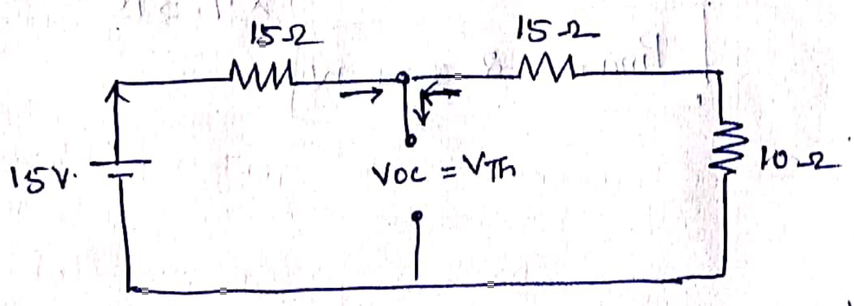
$$P_{max} = \frac{V_{Th}^2}{4 \cdot R_{Th}}$$

Pbm:

Find the value of R_L for maximum power transfer and hence find its maximum power for the given circuit.



step 1 :- To find V_{Th}. Remove R_L and find the voltage across the R_L terminal.



All resistances are connected in series

$R_{eq} = 40\,\Omega$

using ohm's law

$V = IR$

$I = V/R = \frac{15}{40}$

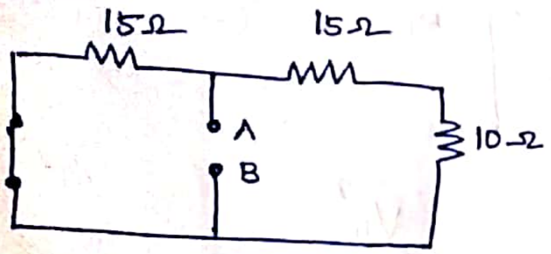
$\boxed{I = 0.375\ A}$

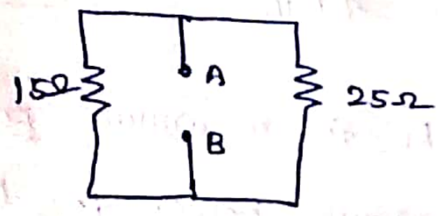
$V_{oc} = V_{Th} = V_{15} + V_{10} = (0.375 \times 15) + (0.375 \times 10)$

$\boxed{V_{Th} = 9.375\ V}$

Step 2:

To find R_{Th}:-





Both are connected in parallel,

$R_{Th} = \frac{25 \times 15}{25 + 15}$

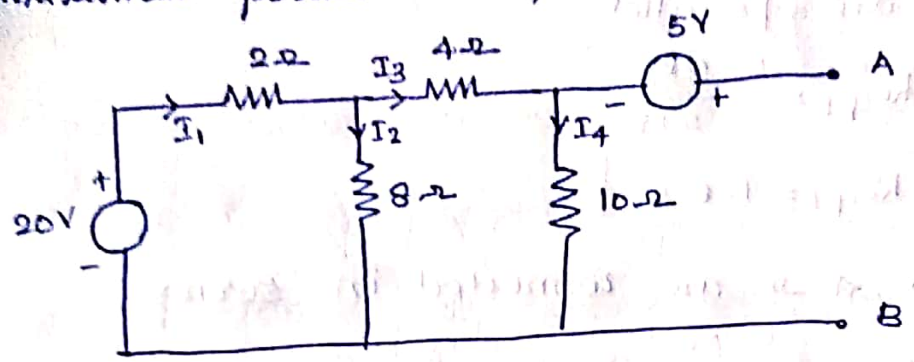
$\boxed{R_{Th} = 9.375\ \Omega}$

Step 3:- Find the maximum power.

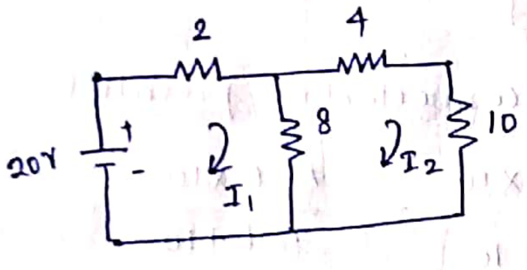
$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{9.375^2}{4 \times 9.375}$

$\boxed{P_{max} = 2.344\ W}$

Determine the value of resistance that may be connected across terminals A and B so that maximum power transfers from the source to the resistance. Also estimate the maximum power transfered to the resistance.



Step 1: To find V_{Th}'s



Apply KVL on loop 1

$$10 I_1 - 8 I_2 = 20 \quad -(1)$$

Apply KVL on loop 2

$$-8 I_1 + 22 I_2 = 0 \quad -(2).$$

$$I_1 = 2.82 \text{ A}$$

$$I_2 = 1.025 \text{ A}.$$

Current flowing through 10Ω resistance = 1.025 A

∴ Voltage across 10Ω = 1.025 × 10

= 10.25 V

Then V_{AB} = 5 + 10.25

$$\boxed{V_{Th} = V_{AB} = 15.25 \text{ V}}$$

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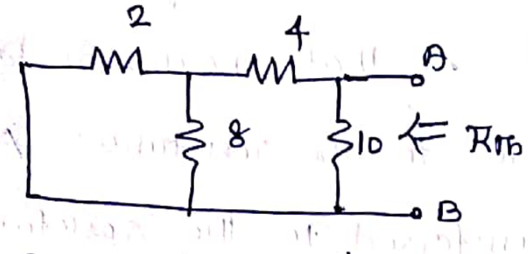
Step 2

To find R_{Th} :-



2Ω & 8Ω are parallel.

$$R_{eq1} = \frac{2\times8}{2+8}$$

$$\boxed{R_{eq1} = 1.6\,\Omega}$$

R_{eq1} & $4\,\Omega$ are connected in series.

$$R_{eq2} = 1.6 + 4$$

$$\boxed{R_{eq2} = 5.6\,\Omega}$$

R_{eq2} & 10Ω are connected in parallel.

$$R_{eq3} = \frac{R_{eq2}\times10}{R_{eq2}+10} = \frac{5.6\times10}{5.6+10}$$

$$\boxed{R_{eq3} = R_{Th} = 3.59\,\Omega}$$

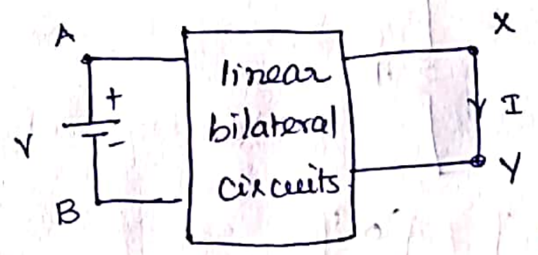
Step 3 :- Find maximum power.

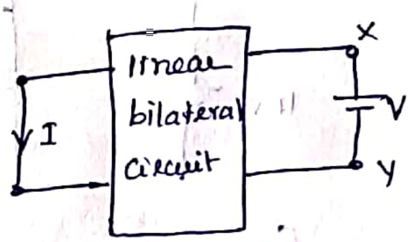
$$P_{max} = \frac{V_{Th}^2}{4\,R_{Th}} = \frac{15.25^2}{4\times3.59}$$

$$\boxed{P_{max} = 16.13\,W}$$

Reciprocity Theorem:

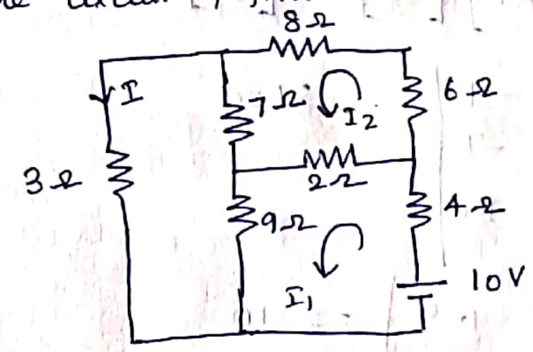
In a linear, bilateral circuit a voltage source V volts in a branch, gives the current I in another branch. The ratio of voltage (V) to current (I) remains constant when the position of V and I are interchanged. This ratio is called transfer resistances. The circuit which satisfy reciprocity theorem, is called reciprocal circuit.





Pbm:

In the circuit, find I and verify reciprocity theorem.



$$[R][I] = [V]$$

$$\begin{bmatrix} 15 & -2 & -9 \\ -2 & 23 & -7 \\ -9 & -7 & 19 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

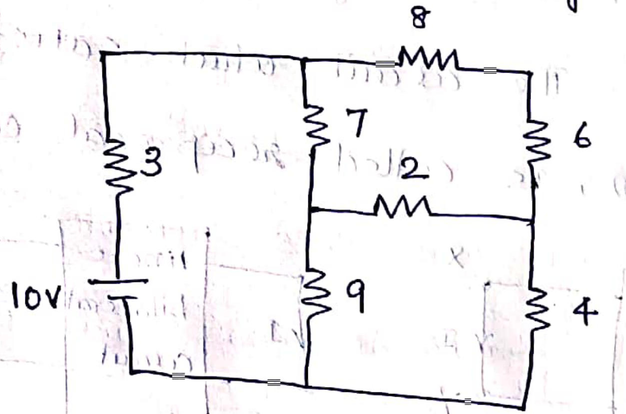
$$\Delta = \begin{vmatrix} 15 & -2 & -9 \\ -2 & 23 & -7 \\ -9 & -7 & 19 \end{vmatrix} = 3629$$

$$\Delta_3 = \begin{vmatrix} 15 & -2 & 10 \\ -2 & 23 & 0 \\ -9 & -7 & 0 \end{vmatrix} = 2210$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{2210}{3629}$$

$$\boxed{I_3 = 0.61\,A}$$

On changing the 10V battery source to the 3Ω resistance branch, circuit will be changed.



$$\begin{bmatrix} 15 & -2 & -9 \\ -2 & 23 & -7 \\ -9 & -7 & 19 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 15 & -2 & -9 \\ -2 & 23 & -7 \\ -9 & -7 & 19 \end{vmatrix} = 3629$$

$$\Delta_1 = \begin{vmatrix} 0 & -2 & -9 \\ 0 & 23 & -7 \\ 10 & -7 & 19 \end{vmatrix} = 2210$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2210}{3629} \qquad \therefore \boxed{I_1 = 0.61\,A}$$

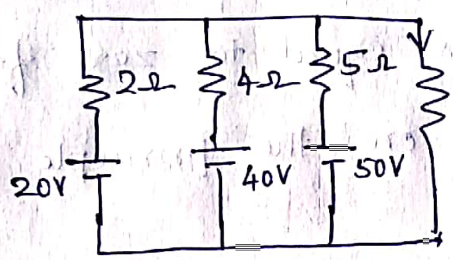
Hence Reciprocity theorem is proved.

Millman's Theorem :-

The theorem states that the voltage across the parallel connections of the circuit includes above one voltage source so it decreases the circuit complexity.

The utility of millman's theorem is that the number of parallel voltage sources can be reduced to one equivalent sources.

Pbm:- Using millman's theorem, find the current through R_L for the circuit



$$R_{eq} = \frac{2 \times 4 \times 5}{2+4+5}$$

$$\boxed{R_{eq} = 1.052\ \Omega}$$

$$V = \frac{E_1 \times Y_1 + E_2 Y_2 + E_3 Y_3}{Y_1 + Y_2 + Y_3}$$

$$= \frac{20 \times 0.5 + 40 \times 0.25 \times 50 \times 0.2}{0.95}$$

$$\boxed{V = 31.57\ V}$$

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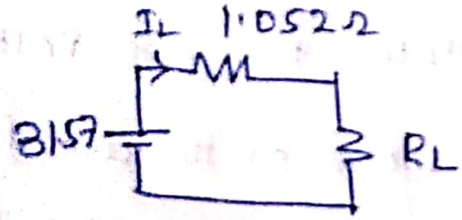
Millman's equivalent circuit



$$I_L = \frac{31.57}{1.052 + 9.4}$$

$$\boxed{I_L = 3.02\ A}$$

Tellegen's Theorem

Theorem states that the summation of instantaneous power consumed by various elements in various branches is equal to zero for any network. Theorem works based on the principle of law of conservation of energy.

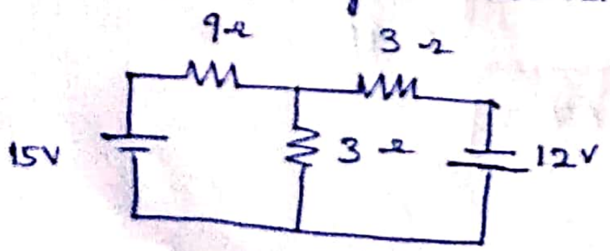
Sum of all instantaneous power

$$P_1 + P_2 + P_3 + \dots P_n = 0$$

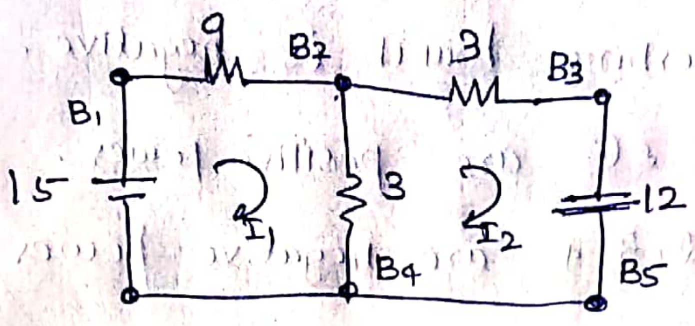
$$P_1 = V_1 I_1 \quad , \quad P_2 = V_2 I_2 \quad \dots \quad P_n = V_n \cdot I_n$$

$$\therefore \sum_{K=1}^{n} P_K = \sum_{K=1}^{n} V_K i_K = 0 .$$

Pbm: Justify Tellegen's Theorem for the below network



Step 1 :- Given circuit has 5 branches.



Apply KVL to the loop 1

$$15 = 12 I_1 - 3 I_2 \quad —①$$

Apply KVL to the loop 2

$$12 = -3 I_1 + 6 I_2 \quad —②$$

Equate eqn ① & ②, we get

$I_1 = 2 A$

$I_2 = 3 A$.

Step 2: Power at each branch.

Branch 1 , $P_1 = V_1 \times I_1 = 15 \times 2 = 30 W$

2 , $P_2 = V_2 \times I_2 = I_1^2 R_1 = 2^2 \times 9 = 36 W$

In branch no 3, we have 2 current values. Total current is $I_3 = 3 - 2 = 1 A$.

Branch 3 , $P_3 = I_3^2 R_2 = 3 \times 1 = 3 W$

4 , $P_4 = R_3 I_2^2 = 3 \times 9 = 27 W$

5 , $P_5 = V I_2 = 12 \times 3 = 36 W$

Step 3:

Power delivering branches, 1 & 5

Power absorbing branches, 2, 3, 4

Step 4: sign of power delivering branch is posit[ive]

sign of power absorbing branch is negative.

branch ① & ⑤ are positive power.

branch 2, 3, 4 are negative power.

Step 5:

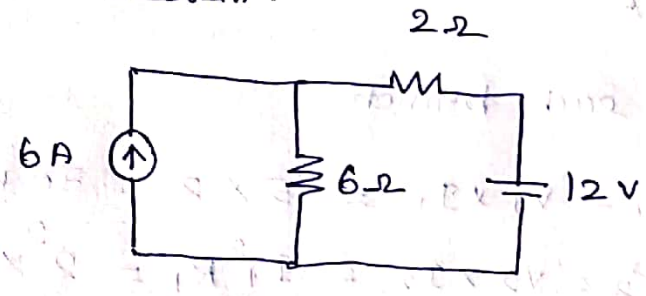
$$P_1 - P_2 - P_3 - P_4 + P_5 = 0$$

$$30 - 36 - 3 - 27 + 36 = 0$$

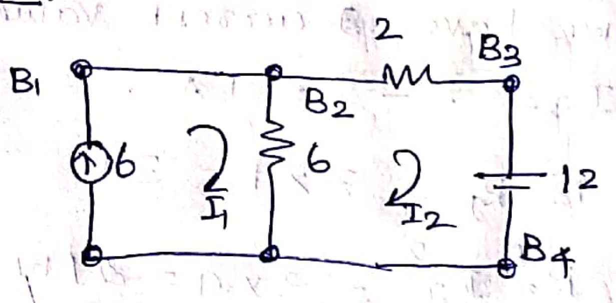
The summation of instantaneous power is zero. Hence the theorem is proved.

Pbm

Find the voltage across 6A current source using Tellegen's theorem.



Step 1:



Apply KVL to loop 2,

$$-12 = 8I_2 - 6I_1$$

From the circuit $\boxed{I_1 = 6\,A}$.

$$\therefore -12 = 8I_2 - 6 \times 6$$

$$\boxed{I_2 = 3A}$$

Step 2:- current through each branch.

Branch 1 , $I_1 = 6A$

2 , $I_{12} = I_1 - I_2 = 6 - 3$

$I_{12} = 3A$

3 , $I_2 = 3A$.

4 , $I_2 = 3A$.

Step 3:- Power at each branch.

$P_1 = I_1 V = 6 \times V$

$P_2 = R_1 I_{12}^2 = 6 \times 9 = 54W$

$P_3 = R_2 I_2^2 = 2 \times 9 = 18W$

$P_4 = V I_2 = -12 \times 3 = -36W$

Step 4:- $P_1 \rightarrow$ positive power

$P_4 \rightarrow$ positive power

$P_2, P_3 \rightarrow$ negative power.

Steps:

$P_1 - P_2 - P_3 + P_4 = 0 W$.

$P_1 - 54 - 18 - 36 = 0W$.

$P_1 = 54 + 18 + 36$

$\boxed{P_1 = 108 W}$

$\therefore$ We know that $P_1 = V_1 I_1$

$\therefore V_1 = P_1 / I_1 = 108/6$

$\boxed{V_1 = 18 V}$

Transient Response Analysis

Circuit Analysis Using Laplace Transform:

Laplace Transform of $f(t)$ is given by

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)\, e^{-st}\, dt + f(0).$$

Inverse Laplace Transform,

$$f(t) = \mathcal{L}^{-1}[F(s)].$$

Initial value theorem,

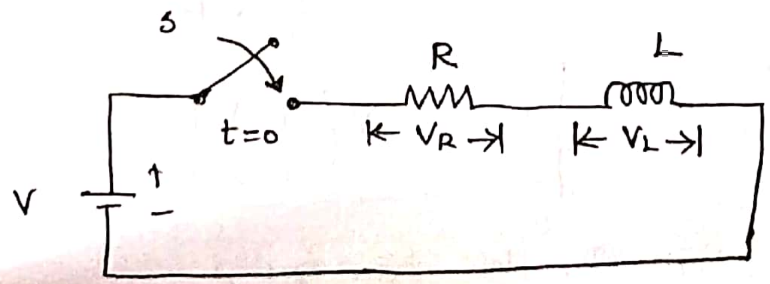
$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s.[F(s)]$$

Final value theorem,

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s.(F(s)]$$

Transient Response of RL Circuit :- (DC or step input)

An RL circuit, which is connected to a DC battery of volts 'V' and a switch 's'. Assum switch 's' is closed at time, $t=0$ and also assume the initial current in the circuit is zero.



By applying KVL to the loop, we get

$$V = V_R + V_L$$

$$V = iR + L \cdot \frac{di}{dt} \quad \text{——— (1)}$$

Taking Laplace transform on both sides.

$$\frac{V}{s} = R\,I(s) + L\left[s \cdot I(s) - i(0)\right]$$

$i(0) \rightarrow$ initial current before switch is closed.

But we assumed $i(0)$ is zero.

$$\therefore \quad R\,I(s) + Ls\,I(s) = \frac{V}{s}$$

$$I(s)\left[R + sL\right] = \frac{V}{s}.$$

$$I(s) = \frac{V}{s\left[R+sL\right]} \quad \text{——— (2)}.$$

By using partial fraction method,

$$I(s) = \frac{V}{s\left[R+sL\right]} = \frac{A}{s} + \frac{B}{R+sL} \quad \text{— (3)}.$$

$$V = A\left[R+sL\right] + Bs$$

At $s = 0$, then $V = A \cdot R$

$$\boxed{A = \frac{V}{R}} \quad \text{——— (4)}$$

At $s = -\frac{R}{L}$, then $V = -\frac{R}{L} \times B$.

$$\therefore \quad \boxed{B = -V\left(\frac{L}{R}\right)} \quad \text{——— (5)}.$$

sub (A), (B) in eqn (3), we can get,

$$I(s) = \frac{V/R}{s} + \frac{-V(L/R)}{R+sL}$$

$$I(s) = \frac{V}{Rs} - \frac{VL}{R\left[R+sL\right]}$$

Simplify the eqn, we get

$$I(s) = \frac{V}{R}\left(\frac{1}{s}\right) - \frac{V}{R} \times \frac{1}{(s + R/L)}$$

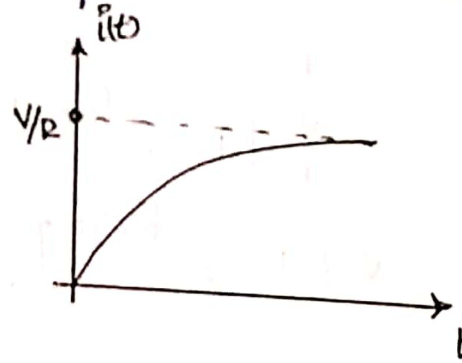
Taking Inverse Laplace Transform,

$$L^{-1}[I(s)] = \frac{V}{R} L^{-1}[1/s] - \frac{V}{R}\left[L^{-1}\left[\frac{1}{s+R/L}\right]\right]$$

$$i(t) = \frac{V}{R} - \frac{V}{R} e^{-(R/L)t}$$

$$\boxed{i(t) = \frac{V}{R}\left[1 - e^{-(R/L)t}\right]}$$

The above equation can be drawns,



The current is rising exponentially.

Final value current can be found by sub. $t \to \infty$

$$i(t)\Big|_{at\ t\to\infty} = \frac{V}{R}\left[1 - e^{-(R/L)\infty}\right]$$

$$\therefore \boxed{i(t)\Big|_{t=\infty} = \frac{V}{R}}$$

then

$$i(t)\Big|_{at\ t=L/R} = \frac{V}{R}\left[1 - e^{-R/L \times \frac{L}{R}}\right]$$

$$= \frac{V}{R}\left[1 - e^{-1}\right]$$

$$= \frac{V}{R}\left[1 - 0.368\right]$$

$$i(t) = 0.632 \times \frac{V}{R}$$

$\therefore$ $i(t) = 63.2\%$ of steady state or final value.

RL circuit Time constant:

The time constant of RL series circuit is defined as the time during which the current increases to 63.2% of its steady state value.

$$\text{Time constant} = \tau = \frac{L}{R}$$

Transient voltage across Resistor:-

$$V_R = i \cdot R$$

$$= \frac{V}{R}\left[1 - e^{-(R/L)t}\right] \cdot R$$

$$V_R = V\left[1 - e^{-(R/L)t}\right].$$

Transient voltage across inductor is

$$V_L = L \cdot \frac{di}{dt}$$

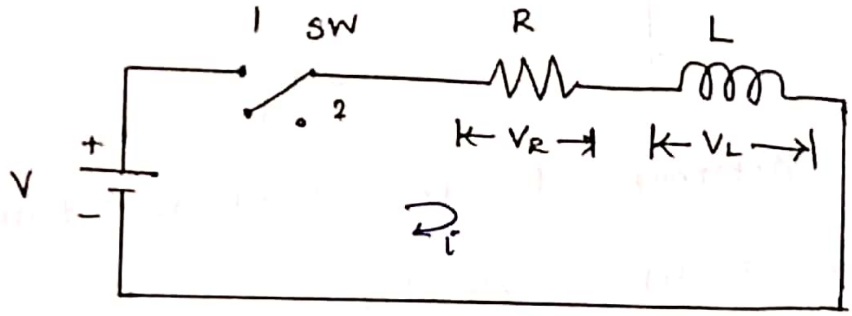
$$= L \times \frac{d}{dt}\left[\frac{V}{R}\left(1 - e^{-(R/L)t}\right)\right]$$

$$= L \times \frac{V}{R}\left[\left(-\frac{R}{L}\right)\left(-e^{-(R/L)t}\right)\right]$$

$$\therefore \boxed{V_L = V \cdot e^{-(R/L)t}}$$

RL Decaying Transient Circuit

consider the circuit, with switch sw is previously in position 1, till steady state conditions are reached. During the switch is in position 1, current in the circuit is $i = \frac{V}{R}$. At time $t=0$, the switch is turned to position 2 which makes the circuit a short circuit.



The voltage equation for the above circuit.

$$V_R + V_L = 0.$$

$$iR + L\frac{di}{dt} = 0.$$

Taking Laplace Transform on both sides.

$$I(s) \cdot R + L\left[s \cdot I(s) - i(0)\right]$$

$i(0) \rightarrow$ current during initial condition,

$$i(0) = \frac{V}{R}$$

$$I(s)\left[R + sL\right] = \frac{V \cdot L}{R}$$

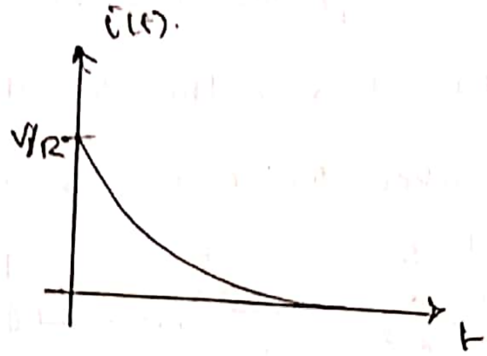
$$I(s) = \frac{V \cdot L}{R}\left[\frac{1}{R + sL}\right]$$

$$I(s) = \frac{V}{R}\left[\frac{1}{s + R/L}\right]$$

Taking inverse laplace Transform,

$$i(t) = L^{-1}[I(s)] = \frac{V}{R} L^{-1}\left[\frac{1}{S+R/L}\right]$$

$$i(t) = \frac{V}{R} \cdot e^{(-R/L)t}$$



To determine the current at any time $t = t_1$, after the switch is closed,

$$i(t)\Big|_{t=\frac{L}{R}} = \frac{V}{R} e^{-\frac{R}{L} \times \frac{L}{R}}$$

$$= \frac{V}{R} e^{-1}$$

$$i(t)\Big|_{t=L/R} = 0.368 \times \frac{V}{R}$$

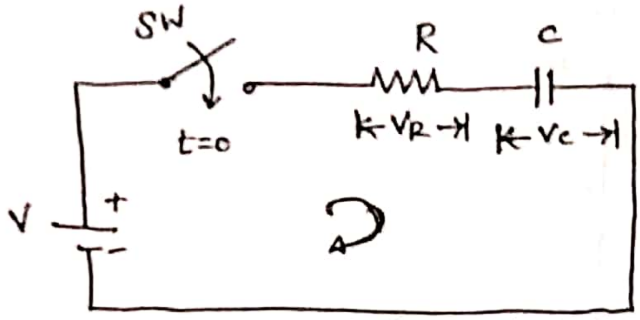
= 36.8% of initial value.

Time constant of RL decay transient circuit is defined as the time during the current decays or decrease to 36.8% of the initial value.

— x —

Transient Response of RC Circuit

An RC circuit, which is connected to a DC battery of V volts and switch, SW. Assume the switch is closed at time, $t=0$ and also assume the initial charge and initial voltage in the capacitor is zero.



The voltage equation for the above circuit is

$$V_R + V_C = V$$

$$\Rightarrow iR + \left[\frac{1}{C}\int i\,dt + V_0\right] = V$$

$$iR + \frac{1}{C}\int i\,dt = V \qquad \because V_0 = 0$$

Taking Laplace transform on both sides,

$$I(s) \cdot R + \frac{1}{C}\frac{I(s)}{s} = \frac{V}{s}$$

$$I(s)\left[R + \frac{1}{Cs}\right] = \frac{V}{s}.$$

$$I(s) = \frac{V}{s\left[R + \frac{1}{Cs}\right]}$$

$$\therefore I(s) = \frac{V}{R\left[s + \frac{1}{RC}\right]}$$

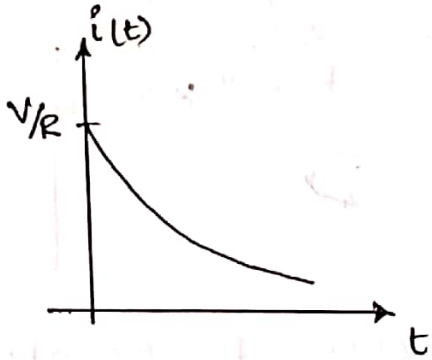
Taking inverse Laplace transform on both sides

$$\boxed{i(t) = \frac{V}{R} e^{-t/RC}}$$

At $t = 0$, $i(t) = \frac{V}{R}$

At $t = \infty$, $i(t) = 0$.

The transient response of RC transient circuit can be drawn.



After a particular period, the current reaches zero, as the capacitor voltage increases and becomes equal to applied voltage, gradually. Consider a time to be RC seconds.

$$i(t)\Big|_{t=RC} = \frac{V}{R} e^{-RC/RC}$$

$$= \frac{V}{R} \cdot e^{-1}$$

$$i(t)\Big|_{t=RC} = 0.368 \times \frac{V}{R}$$

$$= 36.8\% \text{ of initial current.}$$

The time constant of RC series circuit is defined as the time during which the current reaches 36.8% of its initial value.

voltage across the resistor, $V_R = iR$

$$V_R = \left[\frac{V}{R} \cdot e^{-t/RC}\right] \cdot R$$

$$\boxed{V_R = V \cdot e^{-t/RC}}$$

Voltage across the capacitor, $V_C = \frac{1}{C}\int_0^t i(t)\,dt$

$$V_C = \frac{1}{C}\int_0^t \frac{V}{R} e^{-t/RC}\,dt$$

$$= \frac{V}{RC}\int_0^t e^{-t/RC}\,dt$$

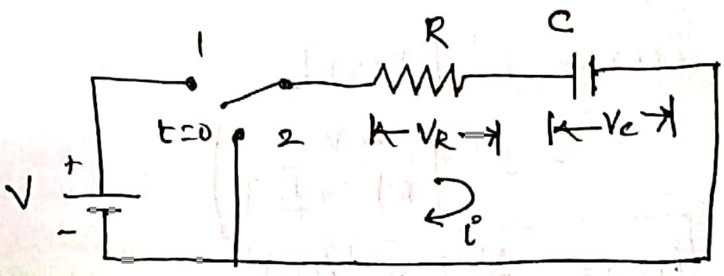
$$= \frac{V}{RC}\left[-\frac{e^{-t/RC}}{1/RC}\right]_0^t$$

$$= \frac{V \times RC}{RC}\left[-e^{-t/RC} - 1\right]$$

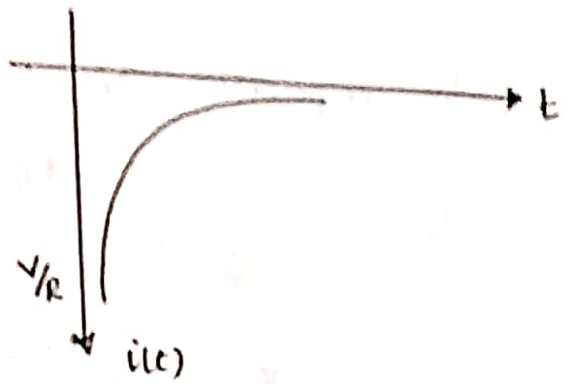
$$\therefore \boxed{V_C = V\left[1 - e^{-t/RC}\right]}$$

RC Decaying Transient Circuit :-

Consider the circuit, which with switch SW is previously in position 1 for sufficient time till steady state conditions are reached and t=0 moved to position 2.



Before the switch is moved to position 2, the capacitor gets charged to the voltage V, with the polarity.



The voltage for the given circuit,

$$V_R + V_C = 0$$

$$iR + \frac{1}{C}\int i\,dt + V = 0.$$

Taking Laplace transform on both sides,

$$I(s)\,R + \frac{1}{C}\,\frac{I(s)}{s} = 0 - \frac{V}{s}$$

$$I(s)\left[R + \frac{1}{Cs}\right] = -\frac{V}{s}$$

$$I(s) = \frac{-V}{s\left[R + \frac{1}{Cs}\right]}$$

$$I(s) = \frac{-V}{R\left[s + \frac{1}{RC}\right]}$$

Taking Laplace transform,

$$i(t) = L^{-1}\left[I(s)\right]$$

$$= L^{-1}\left[\frac{-V}{R\left[s + \frac{1}{RC}\right]}\right]$$

$$\boxed{i(t) = -\frac{V}{R}\left[e^{-t/RC}\right]}$$

The exponential decay of capacitor's is voltage can be drawn. The time constant of RC decay circuit can be defined as the time during the current reaches to 36.8% of initial value in reversed current direction. ie, during discharging operation of capacitor.

Time constant $\tau = RC$ seconds.

$$i(t)\Big|_{t=RC} = -\frac{V}{R}\left[e^{-t/RC}\right]$$

$$= -\frac{V}{R}\left[e^{-RC/RC}\right]$$

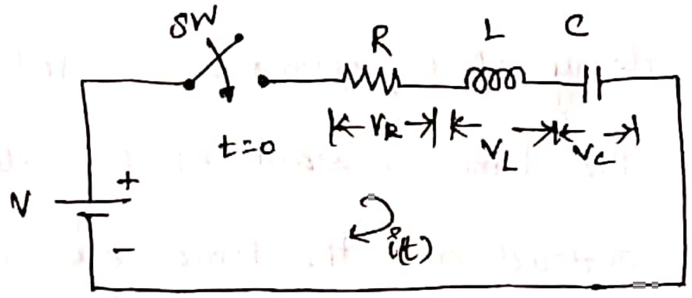
$$= -\frac{V}{R}\left[e^{-1}\right]$$

$$= -\frac{V}{R}\left[0.368\right]$$

$i(t) = -36.8\%$ of initial value.

Transient Response of RLC Circuit :-

An RLC circuit, which is consisting of DC battery and a switch SW, with resistor, inductor and capacitor. Assume the initial current in the circuit to be zero and also the initial voltage of capacitor to be zero.



The voltage equation for the above circuit can be written with the use of KVL as,

$$V_R + V_L + V_C = 0.$$

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0.$$

Taking Laplace Transform,

$$I(S).\,R + L\left[s\,I(S) - i(0)\right] + \frac{1}{C}\,\frac{I(S)}{S} = \frac{V}{S}$$

$$R.\,I(S) + L\left[S\,I(S)\right] + \frac{1}{C}\,\frac{I(S)}{S} = \frac{V}{S}$$

$$I(S)\left[R + SL + \frac{1}{SC}\right] = \frac{V}{S}$$

$$I(S) = \frac{V}{S\left[R + SL + \frac{1}{SC}\right]}$$

$$= \left[\frac{V}{S^2L + SR + \frac{1}{C}}\right]$$

$$I(S) = \frac{V/L}{S^2 + S\frac{R}{L} + \frac{1}{LC}}$$

The roots of the denominator polynomial will be

$$S = \frac{-(R/L) \pm \sqrt{(R/L)^2 - \left(4\times 1\times \frac{1}{LC}\right)}}{2\times 1}$$

$$S = -\left(R/2L\right) \pm \sqrt{\left(R/2L\right)^2 - \left(\frac{1}{LC}\right)}$$

$\therefore \quad s = \alpha \pm \beta$

$$\alpha = -R/2L \quad , \quad \beta = \sqrt{(R/2L)^2 - \left(\frac{1}{LC}\right)}$$

Case (i) :- Roots are real and distinct

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

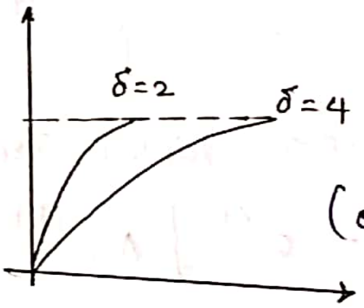
then $S_1 = \alpha + \beta \quad , \quad S_2 = \alpha - \beta$

so $$I(S) = \frac{A}{S - (\alpha+\beta)} + \frac{B}{S - (\alpha - \beta)}$$

Taking Inverse Laplace transform,

$$i(t) = A\, e^{(\alpha+\beta)t} + B\, e^{(\alpha-\beta)t}$$

$$i(t) = e^{\alpha t}\left[A e^{\beta t} + B e^{-\beta t}\right]$$



(over damped)

Case (ii) :- Roots are real,

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} = 0$$

$$\Rightarrow \left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

Then the roots are, $(s-\alpha)^2 \cdot 4\,(s-\alpha)$

$$I(S) = \frac{A}{(s-\alpha)^2} + \frac{B}{(s-\alpha)}$$

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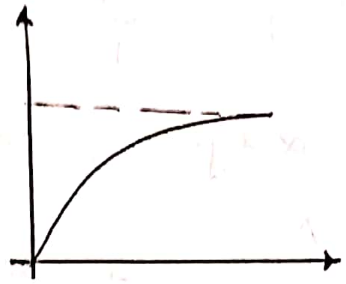
Taking Inverse Laplace Transform, we get.

$$i(t) = At\, e^{\alpha t} + B e^{\alpha t}$$

$$i(t) = e^{\alpha t}(At + B)$$

It is the response of critically damped current.



case (iii) Roots are complex :-

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

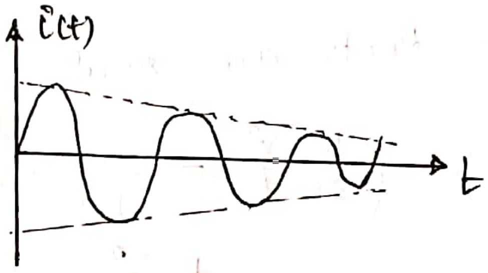
The roots are $(\alpha + j\beta)$ & $(\alpha - j\beta)$

So

$$I(s) = \frac{A}{s-(\alpha+j\beta)} + \frac{B}{s-(\alpha-j\beta)}.$$

Taking inverse Laplace Transform,

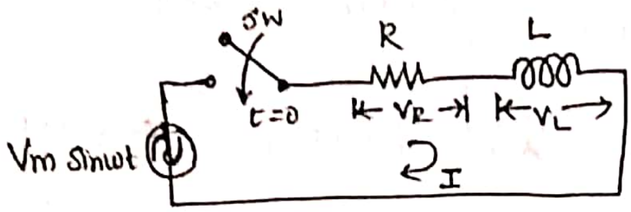
$$i(t) = e^{\alpha t}\left[A e^{j\beta t} + B e^{-j\beta t}\right].$$



The shape of the response say that the current is as oscillatory current, also it is decaying continuously as $\alpha = -\left(\frac{R}{2L}\right)$ is always negative.

Transient Response for AC Circuits

Transient Response of RL Circuit :



consider an RL transient circuit with AC source of Em Sinωt . The switch is closed at t=0 , Assume all initial conditions to be zero .

The voltage equation for the above loop is ,

$$V_R + V_L + V_C = V_m \sin\omega t$$

$$Ri + L\frac{di}{dt} = V_m \sin\omega t$$

Taking Laplace Transform,

$$R\,I(s) + Ls\,I(s) = V_m \cdot \frac{\omega}{s^2+\omega^2}$$

$$I(s)\,[R+Ls] = V_m \cdot \frac{\omega}{s^2+\omega^2}$$

$$I(s) = \frac{V_m \cdot \omega}{(s^2+\omega^2)\,[R+Ls]}$$

$$I(s) = \frac{V_m \cdot \omega}{L\,[s+R/L]\,[s^2+\omega^2]}$$

$$I(s) = \frac{V_m \cdot \omega / L}{[s+R/L]\,[s+j\omega]\,[s-j\omega]}$$

By using partial fraction

$$\frac{V_m \cdot \omega/L}{(s+R/L)(s+j\omega)(s-j\omega)} = \frac{A}{s+R/L} + \frac{B}{s+j\omega} + \frac{C}{s-j\omega}$$

$$V_m \cdot \omega/L = A(s+j\omega)(s-j\omega) + B(s+R/L)(s-j\omega)$$
$$+ C(s+R/L)(s+j\omega).$$

At $s = -j\omega$,

$$\frac{V_m \cdot \omega}{L} = B(s+R/L)(-j\omega - j\omega).$$

$$\frac{V_m \cdot \omega}{L} = B\left[\frac{-j\omega L + R}{L}\right]\left[-2j\omega\right]$$

$$V_m \cdot \omega = B\left[-2\omega^2 L - 2j\omega R\right]$$

$$V_m \cdot \omega = B\left[-2\omega\right]\left[jR + \omega L\right]$$

$$B = -\frac{V_m}{2}\left[\frac{1}{\omega L + jR}\right]$$

$$= -\frac{V_m}{2} \times \frac{1}{\omega L + jR}\left[\frac{\omega L - jR}{\omega L - jR}\right]$$

$$\boxed{B = -\frac{V_m}{2}\left[\frac{\omega L - jR}{R^2 + \omega^2 L^2}\right]}$$

At $s = j\omega$,

$$\frac{V_m \cdot \omega}{L} = C\left[j\omega + \frac{R}{L}\right](2j\omega)$$

$$\frac{V_m \cdot \omega}{L} = C\left[\frac{j\omega L + R}{L}\right] \cdot (2j\omega)$$

At $s = j\omega$ $\quad V_m.\omega \; (j\omega) \; (\quad)$

$$V_m.\omega = C\left[-2\omega^2 L + 2j\omega R\right]$$

$$V_m.\omega = -2\omega C\left[\omega L - jR\right]$$

$$C = -\frac{V_m}{2}\left[\frac{1}{\omega L - jR}\right] \times \frac{\omega L + jR}{\omega L + jR}$$

$$\boxed{C = -\frac{V_m}{2}\left(\frac{\omega L + jR}{R^2 + \omega^2 L^2}\right)}$$

At $s = -\frac{R}{L}$,

$$\frac{V_m.\omega}{L} = A\left[-\frac{R}{L} + j\omega\right]\left[-\frac{R}{L} - j\omega\right]$$

$$\frac{V_m.\omega}{L} = \frac{A}{L^2}\left(-R + j\omega L\right)\left(-R - j\omega L\right)$$

$$V_m.\omega L = A\left(R^2 - j\omega LR - j^2\omega^2 L^2 + j\omega LR\right)$$

$$V_m.\omega L = A\left(R^2 + \omega^2 L^2\right)$$

$$\boxed{A = \frac{V_m.\omega L}{R^2 + \omega^2 L^2}}$$

Substituting the values of A, B and C in I(s) equation,

$$I(s) = \frac{\left(\frac{V_m \omega L}{R^2 + \omega^2 L^2}\right)}{s + R/L} - \frac{\frac{V_m}{2}\left(\frac{\omega L - jR}{R^2 + \omega^2 L^2}\right)}{s + j\omega} - \frac{\frac{V_m}{2}\left(\frac{\omega L + jR}{R^2 + \omega^2 L^2}\right)}{s - j\omega}$$

Taking Laplace Transform,

$$i(t) = \frac{V_m.\omega L}{R^2 + \omega^2 L^2} e^{-(R/L)t} - \frac{V_m}{2}\left(\frac{\omega L - jR}{R^2 + \omega^2 L^2}\right) e^{-j\omega t} - \frac{V_m}{2}\left(\frac{\omega L + jR}{R^2 + \omega^2 L^2}\right) e^{j\omega t}$$

$$i(t) = \left(\frac{V_m.\omega L}{R^2+\omega^2L^2}\right) e^{-(R/L)t} - \frac{V_m}{2(R^2+\omega^2L^2)}\left\{[\omega L - jR].e^{-j\omega t} + [\omega L + jR].e^{j\omega t}\right\}$$

$$i(t) = \frac{V_m.\omega L}{R^2+\omega^2L^2} e^{-(R/L)t} - \frac{V_m}{2(R^2+\omega^2L^2)}\left[\begin{array}{l}(\omega L - jR)(\cos\omega t - j\sin\omega t) + \\ (\omega L + jR)(\cos\omega t + j\sin\omega t)\end{array}\right]$$

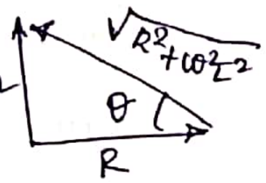
$$i(t) = \frac{V_m.\omega L}{R^2+\omega^2L^2} e^{-(R/L)t} - \frac{V_m}{2(R^2+\omega^2L^2)}\left[\begin{array}{l}\omega L\cos\omega t + j^2R\sin\omega t - \\ j\omega L\sin\omega t - jR\cos\omega t + \\ \omega L\cos\omega t + j\omega L\sin\omega t + \\ j\omega\cos\omega t + j^2R\sin\omega t\end{array}\right]$$

$$\therefore i(t) = \frac{V_m.\omega L}{R^2+\omega^2L^2} e^{-(R/L)t} - \frac{V_m}{2(R^2+\omega^2L^2)} \times \left[2\omega L\cos\omega t + 2j^2R\sin\omega t\right]$$

$$i(t) = \frac{V_m.\omega L}{R^2+\omega^2L^2}.e^{-(R/L)t} - \frac{V_m}{R^2+\omega^2L^2}\left[\omega L\cos\omega t - R\sin\omega t\right]$$

$$i(t) = \frac{V_m}{R^2+\omega^2L^2}\left[\omega L\, e^{-(R/L)t} - \omega L\cos\omega t + R\sin\omega t\right]$$

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left[\frac{\omega L}{\sqrt{R^2+\omega^2L^2}} e^{-(R/L)t} - \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}\cos\omega t + \frac{R}{\sqrt{R^2+\omega^2L^2}}\sin\omega t\right]$$



from diagram,

$$\sin\theta = \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}, \quad \cos\theta = \frac{R}{\sqrt{R^2+\omega^2L^2}}$$

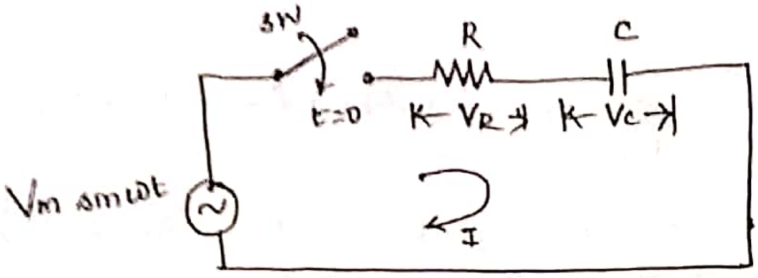
$$\therefore i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left[e^{-(R/L)t}.\sin\theta - \sin\theta\cos\omega t + \cos\theta\sin\omega t\right]$$

$$\boxed{\therefore i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left[e^{-(R/L)t}\sin\theta + \sin(\omega t - \theta)\right]}$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right), \quad \sin\theta = \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}.$$

Transient Response of RC Circuit

Consider an RC circuit, with AC source of voltage $V_m \sin\omega t$. Assume switch is closed at $t=0$ and all initial conditions are zero.



Apply KVL, we can write the voltage equation as,

$$V_m \sin\omega t = V_R + V_C$$

$$V_m \sin\omega t = iR + \frac{1}{C}\int i\,dt$$

Taking Laplace Transfer,

$$V_m\left[\frac{\omega}{s^2+\omega^2}\right] = I(s)\,R + \frac{1}{C}\,\frac{I(s)}{s}$$

$$I(s)\left[R + \frac{1}{sC}\right] = V_m\left[\frac{\omega}{s^2+\omega^2}\right]$$

$$I(s) = \frac{V_m\,.\,\omega\,.\,sC}{(s^2+\omega^2)(sRC+1)}$$

$$I(s) = \frac{V_m\,.\,\omega\,.\,C\,.\,s}{RC\left[s+\frac{1}{RC}\right](s^2+\omega^2)}$$

$$I(s) = \frac{V_m\,.\,\omega\,.\,s/R}{\left(s+\frac{1}{RC}\right)(s+j\omega)(s-j\omega)}$$

By using partial fraction,

$$\frac{V_m\,.\,\omega\, s/R}{\left(s-\frac{1}{RC}\right)(s+j\omega)(s-j\omega)} = \frac{X}{s+\frac{1}{RC}} + \frac{Y}{(s+j\omega)} + \frac{Z}{s-j\omega}$$

$$\frac{V_m \omega s}{R} = X(s+j\omega)(s-j\omega) + Y\left(s+\frac{1}{RC}\right)(s-j\omega) + Z\left(s+\frac{1}{RC}\right)(s+j\omega)$$

At $s = -\frac{1}{RC}$,

$$\frac{V_m.\omega s}{R} = X\left(\frac{-1}{RC}+j\omega\right)\left(\frac{-1}{RC}-j\omega\right)$$

$$\frac{V_m.\omega\left(-\frac{1}{RC}\right)}{R} = \frac{X}{R^2C^2}(-1+j\omega RC)(-1-j\omega RC)$$

$$-\frac{V_m.\omega}{R^2C} = \frac{X}{R^2C^2}\left(1+j\omega RC - j\omega RC - j^2(\omega RC)^2\right)$$

$$-V_m.\omega C = X\left[1+\omega^2R^2C^2\right].$$

$$\boxed{X = \frac{-V_m.\omega C}{[1+\omega^2R^2C^2]}}$$

At $s = -j\omega$,

$$\frac{V_m.\omega(-j\omega)}{R} = Y\left(-j\omega+\frac{1}{RC}\right)(-j\omega-j\omega)$$

$$\frac{-jV_m\omega^2}{R} = \frac{Y}{RC}(-j\omega RC+1)(-2j\omega)$$

$$\frac{V_m.\omega.C}{2} = Y(1-j\omega RC).$$

$$\boxed{Y = \frac{V_m.\omega.C}{2} \times \left[\frac{1}{1-j\omega RC}\right]}$$

$$Y = \frac{V_m.\omega.C}{2} \times \frac{1}{1-j\omega RC} \times \frac{1+j\omega RC}{1+j\omega RC}$$

$$\therefore \boxed{Y = \frac{V_m.\omega.C}{2} \times \frac{(1+j\omega RC)}{1+\omega^2R^2C^2}}$$

At $s=j\omega$, $\frac{V_m.\omega\,(j\omega)}{R} = z\left(j\omega+\frac{1}{RC}\right)(j\omega+j\omega)$

$$j\frac{V_m\omega^2}{R} = \frac{z}{RC}(1+j\omega RC)(2j\omega)$$

$$\frac{V_m\,\omega C}{2} = z\,(1+j\omega RC)$$

$$z = \frac{V_m.\omega C}{2} \times \frac{1}{1+j\omega RC} \times \frac{(1-j\omega RC)}{(1-j\omega RC)}$$

$$\boxed{z = \frac{V_m.\omega C}{2} \times \frac{(1-j\omega RC)}{(1+\omega^2R^2C^2)}}$$

Substituting x, y, z in $I(s)$ equation, we get

$$I(s) = \frac{\frac{-V_m.\omega C}{1+\omega^2R^2C^2}}{\left(s+\frac{1}{RC}\right)} + \frac{\frac{V_m.\omega C}{2}\left[\frac{1+j\omega RC}{1+\omega^2R^2C^2}\right]}{(s+j\omega)} + \frac{\frac{V_m.\omega.C}{2}\left(\frac{1-j\omega RC}{1+\omega^2R^2C^2}\right)}{(s-j\omega)}$$

$$I(s) = \frac{V_m.\omega C}{1+\omega^2R^2C^2}\left[\frac{-1}{s+\frac{1}{RC}} + \frac{1+j\omega RC}{2(s+j\omega)} + \frac{1-j\omega RC}{2(s-j\omega)}\right]$$

Taking Inverse Laplace Transform.

$$i(t) = \frac{V_m\,\omega C}{1+\omega^2R^2C^2}\left[e^{-t/RC} + \frac{(1+j\omega RC)}{2}.e^{-j\omega t} + \frac{(1-j\omega RC)}{2}e^{j\omega t}\right]$$

$$i(t) = \frac{V_m.\omega.C}{1+\omega^2R^2C^2}\left[e^{-t/RC} + \frac{1+j\omega RC}{2}(\cos\omega t - j\sin\omega t) + \frac{1-j\omega RC}{2}(\cos\omega t + j\sin\omega t)\right]$$

$$i(t) = \frac{V_m.\omega.C}{1+\omega^2R^2C^2}\left[e^{-t/RC} + \left[2+\frac{1}{2}\cos\omega t\right] - \left[2\times j^{-2}\frac{\omega RC}{2}\times\sin\omega t\right]\right]$$

$$i(t) = \frac{V_m\,\omega C}{1+\omega^2R^2C^2}\left[e^{-t/RC} + \cos\omega t + \omega RC\,\sin\omega t\right]$$

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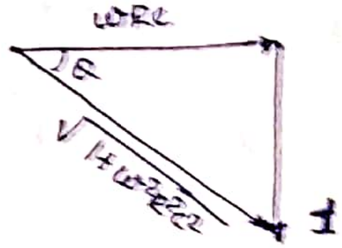
$$i(t) = \frac{-V_m \omega C}{1+\omega^2R^2C^2}\left[\frac{e^{-t/RC}}{\sqrt{1+\omega^2R^2C^2}} - \frac{\cos\omega t}{\sqrt{1+\omega^2R^2C^2}} - \frac{\omega RC \sin\omega t}{\sqrt{1+\omega^2R^2C^2}}\right]$$



considering the above diagram,

$$i(t) = \frac{-V_m \omega C}{\sqrt{1+\omega^2R^2C^2}}\left[e^{-t/RC}\sin\theta - \cos\omega t \sin\theta - \sin\omega t \cos\theta\right]$$

$$= \frac{-V_m \omega C}{\sqrt{1+\omega^2R^2C^2}}\left[e^{-t/RC}\sin\theta - \left[\sin\omega t\cos\theta + \cos\omega t \sin\theta\right]\right]$$

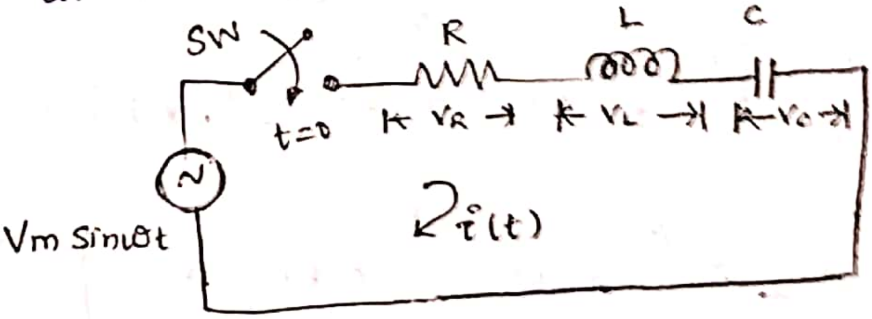
$$i(t) = \frac{-V_m \omega C}{\sqrt{1+\omega^2R^2C^2}} \times \left[e^{-t/RC}\sin\theta - \sin(\omega t+\theta)\right]$$

where $\theta = \tan^{-1}\left(\frac{1}{\omega RC}\right)$

$$\sin\theta = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

Transient response of RLC Circuit :-

consider an RLC transient circuit with AC source of voltage $V_m \sin\omega t$. Assume switch 'S' is closed at $t=0$ and all initial conditions are zero.



Apply KVL, we can write the voltage equation.

$$V_R + V_L + V_C = V_m \sin\omega t$$

$$R\, i(t) + L \cdot \frac{di(t)}{dt} + \frac{1}{C}\int i(t)\, dt = V_m \sin\omega t$$

Taking Laplace transform on both sides.

$$R\, I(s) + L s\, I(s) + \frac{1}{C}\frac{I(s)}{s} = V_m \cdot \frac{\omega}{s^2+\omega^2}$$

$$I(s)\left[R + Ls + \frac{1}{Cs}\right] = \frac{V_m \cdot \omega}{s^2+\omega^2}$$

$$I(s) = \frac{V_m \cdot \omega}{(s^2+\omega^2)\left(R + Ls + \frac{1}{Cs}\right)}$$

$$= \frac{V_m \cdot \omega}{(s^2+\omega^2)\left(\frac{RCs + LCs^2 + 1}{Cs}\right)}$$

$$= \frac{V_m \cdot \omega}{(s^2+\omega^2) \times \frac{L\cancel{C}}{\cancel{C}s}\left[s^2 + \frac{R\cancel{C}}{L\cancel{C}}s + \frac{1}{LC}\right]}$$

$$I(s) = \frac{V_m \cdot \omega \cdot s / L}{(s^2+\omega^2)\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

At $s^2 + \omega^2 = 0$

$$s^2 = -\omega^2$$

$$\boxed{s = \pm\,\omega}$$

The roots are $s_1 = +j\omega$, $s_2 = -j\omega$

At $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$$\therefore\ s = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4 \times 1 \times \frac{1}{LC}}}{2 \times 1}$$

$$S = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Let the roots be, $S = \alpha \pm j\beta$

Then $\alpha = -\frac{R}{2L}$, $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

case (i) Roots are real and distinct.

$$\left(\frac{R}{2L}\right)^2 > \left(\frac{1}{LC}\right)$$

The roots, $S_1 = \alpha + \beta$, $S_2 = \alpha - \beta$

case (ii) Roots are real,

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} = 0.$$

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

The roots are $(S_1 - \alpha)^2$ and $(s_2 - \alpha)$

case (iii) Roots are complex.

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

Then the roots are complex.

$\cancel{\left(\frac{R}{2L}\right)^2}$ $S_1 = (\alpha + j\beta)$ & $S_2 = (\alpha - j\beta)$

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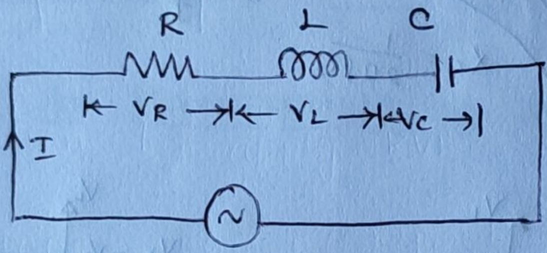
Unit - 4 Electrical Resonance

Condition for Resonance :-

In ac circuit, when $X_L = X_C$, ie a pure resistive circuit, the resonant will occure. $(Z = R)$. At that time in a circuit have a frequency is called Resonant frequency.

Resonance in series RLC circuit :-



consider a RLC circuit, By applying KVL we can write

$$V = V_R + V_L + V_C$$

$V_R \rightarrow$ voltage drop across the resistance

$V_L \rightarrow$ voltage drop across the inductive reactance

$V_C \rightarrow$ voltage drop across the capacitive reactance.

$$V_R = IR \quad , \quad V_L = jI X_L \quad , \quad V_C = -jI X_C$$

$$\therefore \quad V = IR + I \times jX_L + I \times (-jX_C)$$

$$V = I \left[R + jX_L - jX_C \right]$$

$$\frac{V}{I} = R + j(X_L - X_C)$$

Total impedence $Z = \frac{V}{I} = R + j(X_L - X_C)$

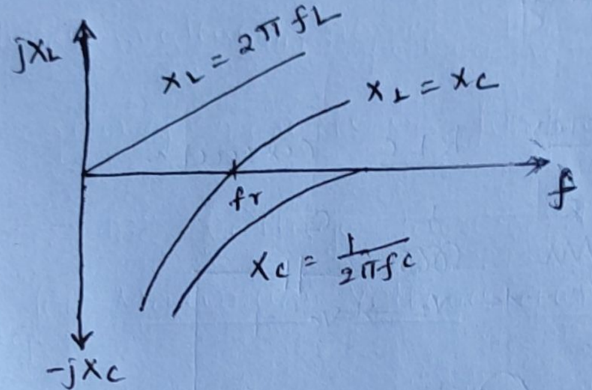
By seeing this impedance equation, it can operated in three cases,

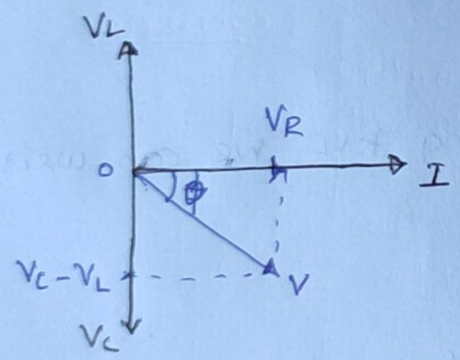
(i) $X_L > X_C$ → inductive circuit

(ii) $X_L < X_C$ → Capacitive circuit

(iii) $X_L = X_C$ → Resonant Circuit ie Resistive circuit

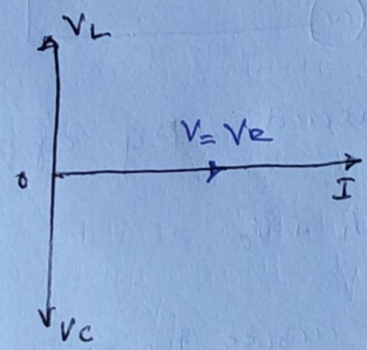
The variations of vector diagram for the three cases,





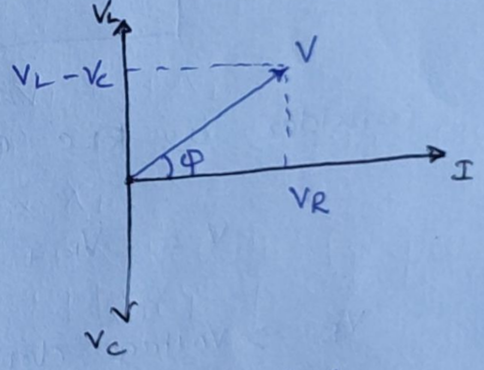
$X_L > X_C$

Lagging power factor



$X_L = X_C$

unity power factor



$X_L < X_C$

Leading power factor.

So that, the RLC series circuit will get into resonance condition at the frequency where inductive reactance is equal to capacitive reactance.

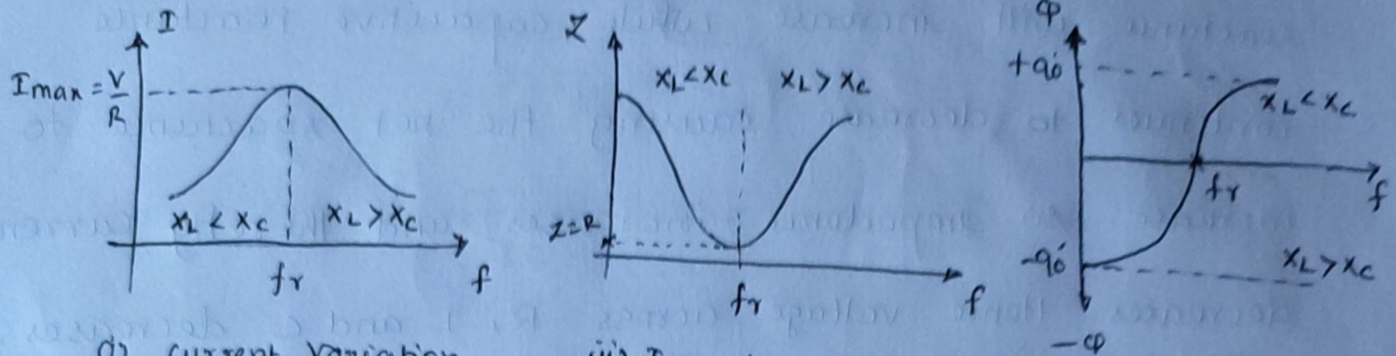
$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C} \Rightarrow 4\pi^2 f^2 LC = 1$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

At resonance, the frequency $\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$ Hz.

The current, impedence and phase angle curves of RLC circuit.



(i) current variation (ii) Impedence variation (iii) Phase angle variation.

Frequencies for Maximum voltage across inductor and capacitor :-

From the impedence diagram, At $f=0$, capacitor acts as open circuit and blocks all the current with frequency. so all the supply voltage will be appearing across the capacitor. when the frequency increases, the capacitive reactance will decrease but inductive reactance will begin to increase from zero. Then the net reactance is given by $(X_C - X_L)$.

As this decreases, total impedence decreases and the current increases. so the voltage across R, L & C increases. At resonant frequency, impedence will be R and hence current will be at its maximum value and

V_R alone will be at its maximum as X_L and X_C cancel each other.

At frequency above resonant frequency, inductive reactance will increase while capacitive reactance continues to decrease causing the net reactance to increase. So impedence increases and thereby current decreases. Hence voltage across R, L and C decreases. When the frequency increases further, the current becomes zero, the voltage across inductor will be supply voltage.

$\therefore$ Voltage across inductor, $V_L = I X_L$

$$V_L = \frac{V}{Z} \times X_L$$

$$V_L = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \times \omega L$$

The condition for V_L to maximum is

$$\frac{dV_L}{d\omega} = 0$$

$$\frac{dV_L}{d\omega} = \frac{d}{d\omega}\left\{ \omega L V \left[R^2 + \left(\omega L - \frac{1}{\omega}\right)^2 \right]^{-1/2} \right\} = 0$$

differenting the above equation with respect to ω and equating it to zero, we get

$$R^2 - \frac{2L}{C} + \frac{2}{\omega^2 C^2} = 0$$

$$\frac{2}{\omega^2 C^2} = \frac{2L}{C} - R^2$$

$$\omega^2 = \frac{2}{C^2} \times \frac{1}{\frac{2L}{C} - R^2}$$

$$\omega^2 = \frac{2}{2LC - R^2C^2}$$

$$\omega = \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}}$$

Then the frequency f_L at which V_L will be maximum is given by,

$$\boxed{f_L = \frac{1}{2\pi\sqrt{LC - \frac{R^2C^2}{2}}}}$$

simplyfy, we get

$$\boxed{f_L = \frac{1}{2\pi\sqrt{LC}} \times \sqrt{\frac{1}{1 - \frac{R^2C^2}{2LC}}}}$$

similarly voltage across capacitor $V_C = I X_C$

$$V_C = \frac{V}{Z} \times X_C$$

$$V_C = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \times \frac{1}{\omega C}$$

The condition for maximum voltage across capacitor is

$$\frac{dV_C}{d\omega} = 0$$

$$\frac{dV_C}{d\omega} = \frac{d}{d\omega}\left[\frac{V}{\omega C}\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{-1/2}\right] = 0.$$

differentiating the above equation with respect to ω and equating it to zero, we get

$$2\omega R^2C^2 + 4\omega^3 L^2 C^2 - 4\omega LC = 0.$$

$$4\omega^3 L^2 C^2 = 4\omega LC - 2\omega R^2 C^2.$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

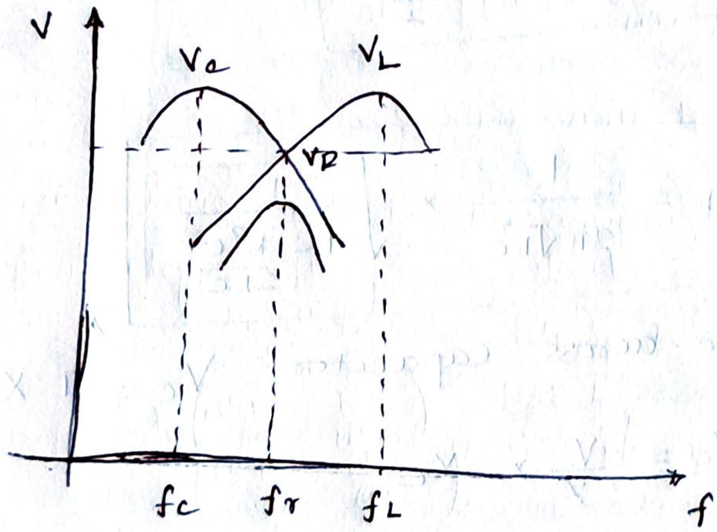
$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

Then the frequency f_c at which V_c will be maximum is

$$f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

Simplifying we get,

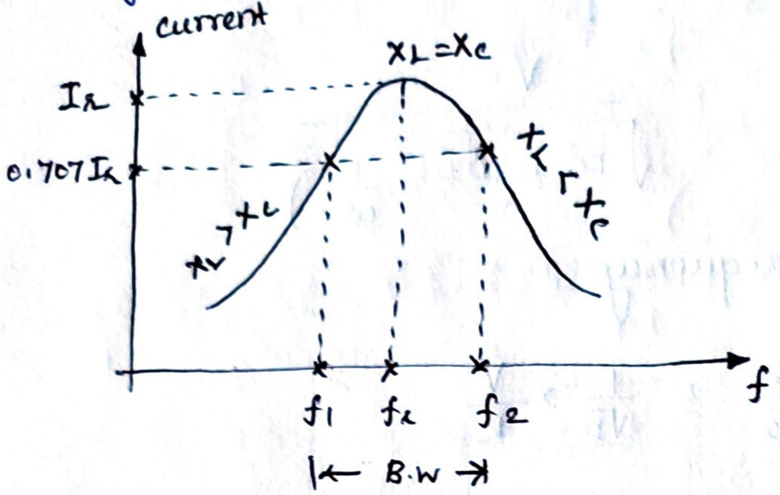
$$\boxed{f_c = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R^2C}{2L}}}$$



Bandwidth:-

The range of frequency for which the response curve is equal to 0.707 times or $\left(\frac{1}{\sqrt{2}}\right)$ time of its maximum value. (or) Bandwidth is defined as the width of resonant curve upto frequency at which the power in the circuit is half of its maximum value.

The frequency at which power in the circuit is half of its maximum value are called half power frequency.



Let I_R be the current, at resonance of a series RLC circuit.

$$I_R = \frac{V}{Z} = \frac{V}{R}$$

This value of I_R at resonance will be maximum since impedance value will be minimum at resonance, then

$$P_R = P_{max} = (I_R)^2 \cdot R$$

At frequency f_1, half power frequency is given by

$$P_1 = \frac{P_R}{2} = \frac{I_R^2 R}{2} = \left(\frac{I_R}{\sqrt{2}}\right)^2 \cdot R = \frac{1}{2} I_R^2 R$$

Similarly, at frequency f_2, half power frequency is given by,

$$P_2 = \frac{P_R}{2} = \frac{I_R^2 R}{2} = \left(\frac{I_R}{\sqrt{2}}\right)^2 R = \frac{1}{2} I_R^2 R$$

The frequency f_1 and f_2 are called to be the lower and upper half power frequencies, respectively.

Then the bandwidth, BW is given by

$$BW = (f_2 - f_1)\ Hz$$

current in series RLC circuit is,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At half power frequency,

$$I = \frac{I_R}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{V}{R}$$

substituting the value of I we get,

$$\frac{1}{\sqrt{2}} \times \frac{V}{R} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}\, R$$

square on both sides,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2 - R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R$$

The above quadratic equation will have two ω values whose frequencies are f_1 and f_2. Now,

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{——— (1)}$$

$$\omega_2 L - \frac{1}{\omega_2 C} = +R \quad \text{——— (2)}$$

Adding the above two equations,

$$\omega_1 L - \frac{1}{\omega_1 C} + \omega_2 L - \frac{1}{\omega_2 C} = 0$$

$$L(\omega_1 + \omega_2) - \left[\frac{1}{\omega_1} + \frac{1}{\omega_2}\right] \times \frac{1}{C} = 0$$

$$L(\omega_1 + \omega_2) = \left[\frac{1}{\omega_1} + \frac{1}{\omega_2}\right] \times \frac{1}{C}$$

$$LC\cancel{(\omega_1 + \omega_2)} = \frac{\cancel{(\omega_1 + \omega_2)}}{\omega_1 \omega_2}$$

$$\boxed{\omega_1 \omega_2 = \frac{1}{LC}} \quad \text{———} \quad (3)$$

But angular frequency at resonant condition is given by

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\omega_1 \times \omega_2 = \omega_r^2$$

$$f_1 \times f_2 = f_r^2$$

$$\therefore \boxed{f_r = \sqrt{f_1 f_2}} \quad \text{———} \quad (4)$$

subtracting the eqn (2) to eqn (1),

$$\omega_2 L - \frac{1}{\omega_2 C} - \left[\omega_1 L - \frac{1}{\omega_1 C}\right] = R - (-R)$$

$$\omega_2 L - \frac{1}{\omega_2 C} - \omega_1 L + \frac{1}{\omega_1 C} = R + R$$

$$L(\omega_2 - \omega_1) - \frac{1}{C}\left[\frac{1}{\omega_2} - \frac{1}{\omega_1}\right] = 2R$$

$$L(\omega_2 - \omega_1) - \frac{1}{C}\left[\frac{\omega_1 - \omega_2}{\omega_1 \omega_2}\right] = 2R$$

$$L(\omega_2-\omega_1)+\frac{1}{C}\frac{(\omega_2-\omega_1)}{\omega_1\omega_2}=2R$$

The above equation devide by L

$$\therefore (\omega_2-\omega_1)+\frac{1}{LC}\frac{(\omega_2-\omega_1)}{\omega_1\omega_2}=\frac{2R}{L}$$

and W.K.T $\omega_1\omega_2=\frac{1}{LC}$

$$\therefore (\omega_2-\omega_1)+\frac{1}{\cancel{LC}}\times \cancel{LC}(\omega_2-\omega_1)=\frac{2R}{L}$$

$$(\omega_2-\omega_1)+(\omega_2-\omega_1)=\frac{2R}{L}$$

$$\cancel{2}(\omega_2-\omega_1)=\frac{\cancel{2}R}{L}$$

$$\omega_2-\omega_1=\frac{R}{L}$$

and $\omega_1=2\pi f_1$, $\omega_2=2\pi f_2$

$$\therefore 2\pi f_2-2\pi f_1=\frac{R}{L}$$

$$2\pi(f_2-f_1)=\frac{R}{L}$$

$$\boxed{f_2-f_1=\frac{R}{2\pi L}}$$

Bandwidth $B.W=f_2-f_1=\frac{R}{2\pi L}$

Also half power frequencies can be represented interms of resonant frequency as,

Lower half power frequency $f_1=f_r-\frac{R}{4\pi L}$

Upper half power frequency $f_2=f_r+\frac{R}{4\pi L}$

x x

Selectivity:-

It is the ratio of bandwidth to the resonant frequency of the circuit.

$$\text{Selectivity} = \frac{\text{Bandwidth}}{\text{Resonant frequency}}$$

$$= \frac{f_2 - f_1}{f_r} = \frac{\frac{R}{2\pi L}}{f_r}$$

$$\text{Selectivity} = \frac{R}{2\pi f_r L} = \frac{R}{\omega_r L}$$

also $\text{selectivity} = \frac{1}{Q}$

$Q \rightarrow$ Quality factor.

Quality factor:-

It is the ratio of the maximum energy stored in the inductor or capacitor to the total energy dissipated per cycle.

$$\text{Quality factor} = 2\pi \times \frac{\text{Maximum Energy stored}}{\text{Energy dissipated per cycle.}}$$

Maximum energy stored in an inductor $= \frac{1}{2} L I_m^2$

Energy dissipated per cycle $= \left(\frac{I_m}{\sqrt{2}}\right)^2 \times RT$

$$Q = 2\pi \times \frac{\frac{1}{\cancel{2}} L \cancel{I_m^2}}{\frac{\cancel{I_m^2}}{\cancel{2}} R \times \frac{1}{f}} \qquad \because T = \frac{1}{f}$$

$$Q = \frac{2\pi f_r L}{R}$$

ie $$Q = \frac{\omega_r L}{R}$$

using capacitance $$Q = \frac{1}{\omega_r CR}$$

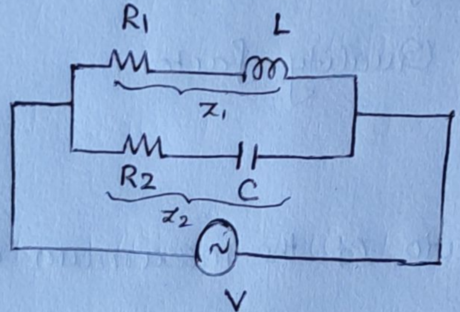
$$Q = \frac{X_L}{R}$$

$$Q = \frac{1}{R} \times \sqrt{\frac{L}{C}}$$

$$Q = \frac{f_r}{f_2 - f_1}$$

Resonance In Parallel RLC Circuit

case (i) :- consider a RLC parallel circuit,



Equivalent impedance of the circuit,

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$= \frac{1}{R_1 + jX_L} + \frac{1}{R_2 - jX_C}$$

$$= \frac{R_1 - jX_L}{(R_1 + jX_L)(R_1 - jX_L)} + \frac{R_2 + jX_C}{(R_2 - jX_C)(R_2 + jX_C)}$$

$$\frac{1}{Z_{eq}} = \frac{R_1}{R_1^2 - (jX_L)^2} + \frac{R_2}{R_2^2 - (jX_C)^2} + \frac{jX_C}{R_2^2 - (jX_C)^2} - \frac{jX_L}{R_1^2 - (jX_L)^2}$$

$$\frac{1}{Z_{eq}} = \left(\frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2}\right) + j\left(\frac{X_C}{R_2^2 + X_C^2} - \frac{X_L}{R_1^2 + X_L^2}\right)$$

At resonant circuit will be purely resistive (ie) net reactance is zero, so

$$\frac{X_C}{R_2^2 + X_C^2} - \frac{X_L}{R_1^2 + X_L^2} = 0$$

$$\frac{X_C}{R_2^2 + X_C^2} = \frac{X_L}{R_1^2 + X_L^2}$$

$$X_C\left[R_1^2 + X_L^2\right] = X_L\left[R_2^2 + X_C^2\right]$$

$$\frac{1}{\omega_r C}\left[R_1^2 + \omega_r^2 L^2\right] = \omega_r L\left[R_2^2 + \frac{1}{\omega_r^2 C^2}\right]$$

$$R_1^2 + \omega_r^2 L^2 = \omega_r^2 LC\left[R_2^2 + \frac{1}{\omega_r^2 C^2}\right]$$

$$R_1^2 + \omega_r^2 L^2 = \omega_r^2 R_2^2 LC + \frac{L}{C}$$

$$\omega_r^2 L^2 - \omega_r^2 R_2^2 LC = \frac{L}{C} - R_1^2$$

$$\omega_r^2 LC\left[\frac{L}{C} - R_2^2\right] = \frac{L}{C} - R_1^2$$

$$\omega_r^2 LC = \frac{\frac{L}{C} - R_1^2}{\frac{L}{C} - R_2^2}$$

$$\omega_r^2 = \frac{1}{LC}\left[\frac{\left(\frac{L}{C} - R_1^2\right)}{\left(\frac{L}{C} - R_2^2\right)}\right]$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}\left[\frac{\frac{L}{C} - R_1^2}{\frac{L}{C} - R_2^2}\right]^{1/2}}$$

Comparision between Series & Parallel Resonance

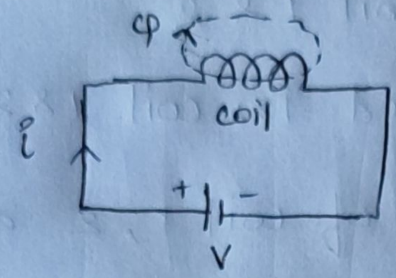
Parameter	Series Resonance Circuit	Parallel Resonance Circuit.
1) Value of impedence at resonance	Minimum	Maximum.
2. Current at resonance	Maximum $= \frac{V}{R}$	Minimum $= V \times \frac{CR}{L}$
3. Effective impedence at resonance	R	$\frac{L}{CR}$
4. Power factor at resonance	unity	unity
5. Resonant frequency	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

x ———— x ———— x

Coupled Circuit

Self inductance :-

consider a coil of N - turns, carrying i current. As the current is flowing through the coil, a flux ϕ will be produced in the coil.



According to faradays law,

induced emf e is directly proportional to rate of change of flux $\left(\frac{d\phi}{dt}\right)$.

$$e \propto \frac{d\phi}{dt}$$

$$\boxed{e = -N\frac{d\phi}{dt}} \quad \text{——— ①}$$

Negative sign indicates that the direction of the emf is opposite to the change in current.

The emf is also proportional to rate of change of current.

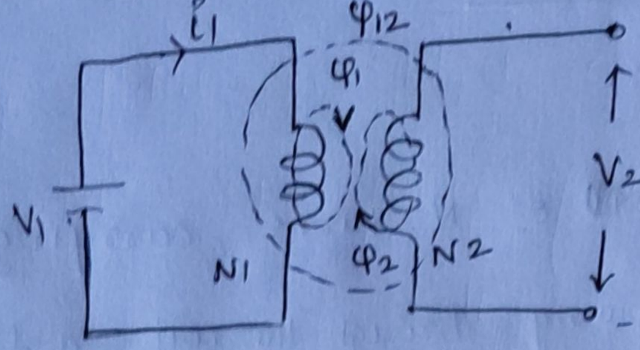
$$e \propto \frac{di}{dt}$$

$$\boxed{e = -L\frac{di}{dt}} \quad \text{— ②}$$

equate ① & ②,

Self inductance of coil, $\boxed{L = \frac{N\phi}{i}}$ Henry.

Mutual inductance:



$N_1, N_2 \rightarrow$ no of coils in coil 1 & 2

$i_1, i_2 \rightarrow$ current in coil 1 & 2

$e_1, e_2 \rightarrow$ Voltage induced in coil 1 & 2

$\varphi_1, \varphi_2, \varphi_{12} \rightarrow$ Self induce of coil 1, 2 and mutual inductance of coil 1 & 2.

If the V_1 voltage is varied, the current i_1 changes, This change in current i_1 produces a flux φ_1. The flux has two parts ie φ_{11} & φ_{12}

$$\varphi_1 = \varphi_{11} + \varphi_{12}$$

$\varphi_{11} \rightarrow$ part of flux φ_1 links with coil 1 only

$\varphi_{12} \rightarrow$ part of φ_1 links with coil 2.

when flux φ_1 changes, then this change produces a voltage in the coil 2 which will be proportional to the rate of change of flux in coil 2.

$$e_2 \propto \frac{d\varphi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\varphi_{12}}{dt} \quad \text{—①}$$

The emf is also proportional to the rate of change of current which creates the change in flux.

$$e_2 \propto \frac{di_1}{dt}$$

$$e_2 = -M\frac{di_1}{dt} \quad \text{--- (2)}$$

equate eqn (1) & (2),

$$\boxed{M = \frac{N_2 \phi_{12}}{i_1}} \quad \text{or} \quad \boxed{M = \frac{N_1 \phi_{21}}{i_2}}$$

M → mutual inductance between coil 1 and 2.

Co-efficient of coupling (or) Magnetic coupling co-efficient

We know that, Mutual inductance is,

$$M = \frac{N_2 \phi_{12}}{i_1} \quad \text{--- (1)} \quad \text{or} \quad M = \frac{N_1 \phi_{21}}{i_2} \quad \text{--- (2)}$$

Coefficient of coupling is defined as the ratio of linkage of flux to the total flux produced.

It is denoted by "k".

$$K = \frac{\phi_{21}}{\phi_2} = \frac{\phi_{12}}{\phi_1}$$

$$\Rightarrow \phi_{12} = k\phi_1 \quad \text{--- (3)}, \quad \phi_{21} = k\phi_2 \quad \text{--- (4)}$$

Substitute the value of ϕ_{12} & ϕ_{21} in the mutual inductance equation we get, multiply eqn (1) & (2),

$$M^2 = \frac{N_2 (k\varphi_1)}{i_1} \times \frac{N_1 (k\varphi_2)}{i_2}$$

$$M^2 = k^2 \left(\frac{N_1\varphi_1}{i_1}\right)\left(\frac{N_2\varphi_2}{i_2}\right)$$

$$M^2 = k^2 L_1 L_2$$

$$M = k\sqrt{L_1 L_2}$$

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

Max. value of $k = 1$, then,

$$\boxed{M_{max} = \sqrt{L_1 L_2}}$$

DOT Rule :-

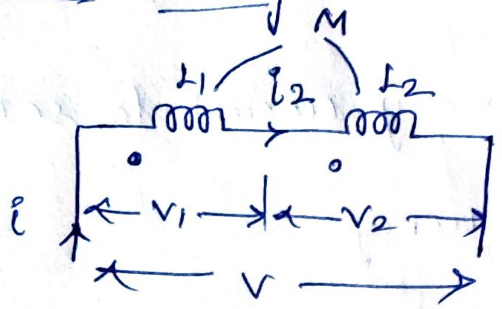
The sign of mutual inductance is based on.

(i) If currents enters or leaves the coils' dotted end in both the coils, then the sign of mutual inductance or mutually induced emf will be positive.

(ii) If one coil current enter in the dotted end, whereas in another coil current leaves the dotted end, then the sign of mutual inductance or mutually induced emf will be negative.

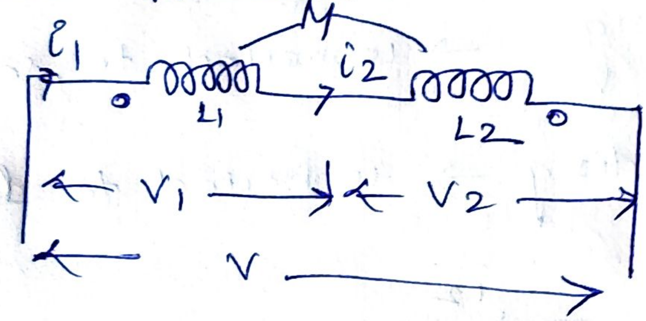
Types of connections in coupled circuits:

(i) Series aiding



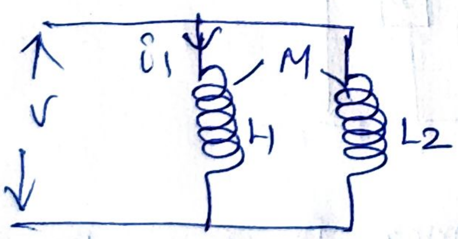
$$\boxed{L_{eff} = L_1 + L_2 + 2M}$$

(ii) Series opposing:-



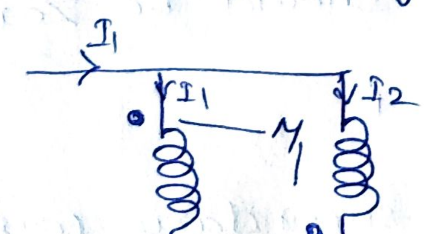
$$\boxed{L_{eff} = L_1 + L_2 - 2M}$$

(iii) Parallel aiding:-



$$\boxed{L_{eff} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

(iv) Parallel opposing:-



$$\boxed{L_{eff} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

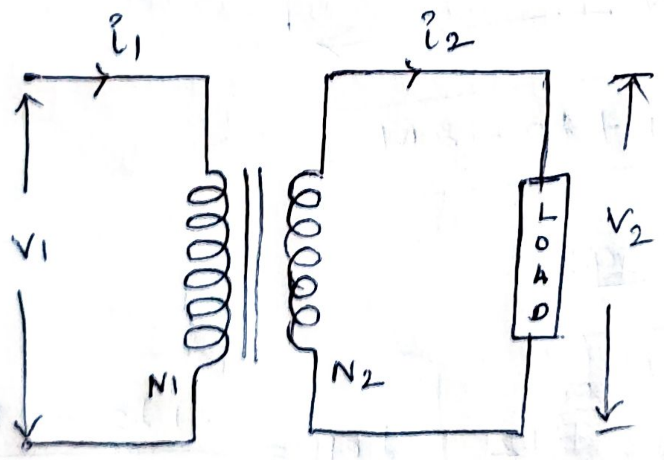
Ideal Transformer:-

Transformer is a static device which will transform electrical energy from primary circuit to secondary circuit, without change in frequency.

Principle → mutual inductance.

Two windings are magnetically coupled.

* primary winding → connected to source
* Secondary winding → connected to load.

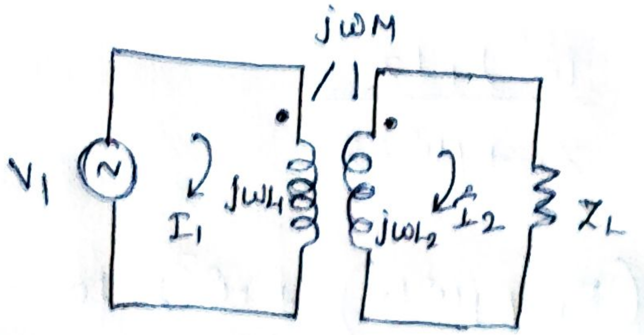


When an ideal transformer is considered,

* Losses in the transformer should be zero.
* No leakage flux ie $k=1$
* Reactance of primary & secondary winding should be large.

Transformation Ratio, $K = \frac{N_2}{N_1}$

consider an ideal transformer,



The voltage eqn of two loops,

$$V_1 = j\omega L_1 I_1 - j\omega M \cdot I_2 \quad \text{——} \; ①$$

$$0 = -j\omega M I_1 + (Z_L + j\omega L_2) I_2 . \quad \text{——} \; ②$$

From ②, $I_2 = \dfrac{j\omega M I_1}{Z_L + j\omega L_2}$ ——— ③

eqn ③ in eqn①, we get -

$$V_1 = j\omega L_1 I_1 - j\omega M \times \frac{j\omega M I_1}{Z_L + j\omega L_2}$$

$$V_1 = j\omega L_1 I_1 - \frac{j^2 \omega^2 M^2 I_1}{Z_L + j\omega L_2}$$

$$V_1 = I_1 \left[j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2} \right]$$

$$Z_{in} = \frac{V_1}{I_1} = j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2} \quad \text{———} \; ④$$

For ideal transformer, the magnetic coupling will be tight and $k=1$, then,

$$M = \sqrt{L_1 L_2}$$

$$M^2 = L_1 L_2 \quad \text{———} \; ⑤$$

Eqn ⑤ in eqn ④, we get.

$$Z_{in} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{Z_1 + j\omega L_2}$$

$$= \frac{j\omega L_1 (Z_1 + j\omega L_2) + \omega^2 L_1 L_2}{Z_1 + j\omega L_2}$$

$$= \frac{j\omega L_1 Z_1 + \cancel{j^2 \omega^2 L_1 L_2} + \cancel{\omega^2 L_1 L_2}}{Z_1 + j\omega L_2}$$

$$Z_{in} = \frac{j\omega L_1 Z_1}{Z_1 + j\omega L_2} \quad \text{———} ⑥$$

By transformer ratio,

$$\frac{N_2^2}{N_1^2} = k^2 = \frac{L_2}{L_1}$$

$$\therefore Z_{in} = \frac{j\omega L_1 Z_L}{Z_L + j\omega k^2 L_1}$$

For ideal transformer, reactance will be very large

$$Z_{in} = \frac{j\omega L_1}{j\omega k^2 L_1} Z_L \qquad \text{as } Z_L \ll L_1$$

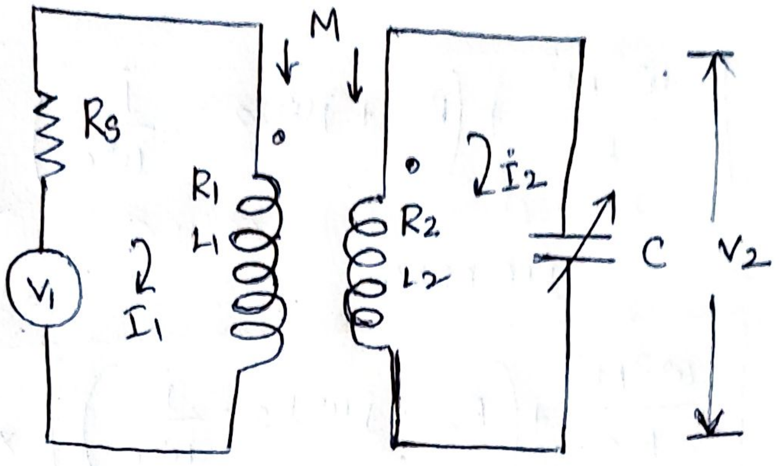
$$Z_{in} = \frac{1}{k^2} Z_L = \left(\frac{N_1^2}{N_2^2}\right) Z_L$$

Similarly

$$\boxed{k = \frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}}$$

Single Tuned Circuit:-

Consider a single tuned circuit, with adjustable capacitor in the secondary current circuit which can be tuned to resonance.



Let R_s → source resistance

R_1, R_2 → Resistance of coil 1 & 2

L_1, L_2 → Self inductance of coil 1 & 2

Assume $R_s >>> R_1$ and $j\omega L_1$. The voltage equation for loop 1,

$$i_1 R_1 - j\omega M i_2 = V_1 \quad \text{——— (1)}$$

Voltage equation for loop 2,

$$-j\omega M i_1 + \left(R_2 + j\omega L_2 - \frac{1}{\omega C}\right) i_2 = 0 \quad \text{——— (2)}$$

From loop 1,

$$i_1 R_s = V_1 + j\omega M i_2$$

$$i_1 = \frac{V_1 + j\omega M i_2}{R_s} \quad \text{——— (3)}$$

Sub. eqn ③ in eqn ②, we get

$$-j\omega M\left(\frac{V_1 + j\omega M i_2}{R_s}\right) + \left(R_2 + j\omega L_2 - \frac{j}{\omega C}\right) i_2 = 0$$

$$-\frac{j\omega M V_1}{R_s} - \frac{j^2\omega^2 M^2 i_2}{R_s} + \left(R_2 + j\omega L_2 - \frac{j}{\omega C}\right) i_2 = 0$$

$$i_2\left[\frac{\omega^2 M^2}{R_s} + \left(R_2 + j\omega L_2 - \frac{j}{\omega C}\right)\right] = \frac{j\omega M V_1}{R_s}$$

$$i_2 = \frac{j\omega M V_1}{\left[\frac{\omega^2 M^2}{R_s} + \left(R_2 + j\omega L_2 - \frac{j}{\omega C}\right)\right] \times R_s} \quad \text{—— ④}$$

Output voltage $V_2 = i_2 \times \frac{1}{j\omega C}$ —— ⑤

Sub eqn ⑤ in eqn ④,

$$i_2 = \frac{1}{j\omega C} \times \frac{1}{R_s} \times \frac{j\omega M V_1}{\frac{\omega^2 M^2}{R_s} + j\left(\omega L_2 - \frac{1}{\omega C}\right)}$$

$$V_2 = \frac{V_1 M}{C} \times \frac{1}{R_s\left[\frac{\omega^2 M^2}{R_s} + R_2 + j\left(\omega L_2 - \frac{1}{\omega C}\right)\right]}$$

This circuit can be tuned to resonance by adjusting the capacitor C. At resonant frequency ω_r

$$\omega_r L_2 = \frac{1}{\omega_r C}$$

output voltage at resonant frequency.

$$V_2 = \frac{V_1 M}{C} \times \frac{1}{R_s R_2 + \omega_r^2 M^2}$$

Secondary current i_2 at resonance condition is

$$i_2 = \frac{j\omega M V_1}{R_s R_2 + \omega_r^2 M^2}$$

The maximum output will be collected and its condition is

$$\frac{dV_2}{dM} = 0$$

$$\frac{d}{dM}\left[\frac{V_1 M}{C\left[R_2 R_s + \omega_r^2 M^2\right]}\right] = 0$$

$$1 - 2M^2\omega_r^2\left[R_s R_2 + \omega_r^2 M^2\right] = 0$$

$$\frac{2M^2\omega_r^2}{R_s R_2 + \omega_r^2 M^2} = 1$$

$$R_s R_2 + \omega_r^2 M^2 = 2M^2\omega_r^2$$

$$R_s R_2 = \omega_r^2 M^2$$

$$\text{and } M = \sqrt{\frac{R_s R_2}{\omega_r^2}}$$

when M is high, then the highest output voltage

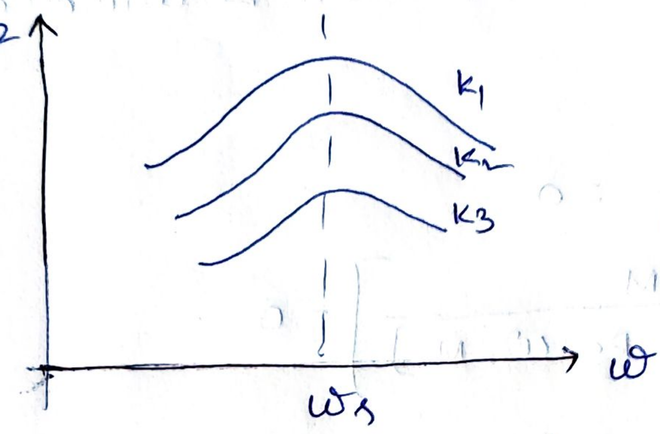
$$V_{2max} = \frac{V_1 \sqrt{R_s R_2}}{\omega_r C\left[R_s R_2 + R_s R_s\right]}$$

$$V_{2\,max} = \frac{V_1}{2\omega_r C\sqrt{R_1 R_2}}$$

The variation of output voltage V_2 with the coefficient of coupling as,

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

$$k_1 > k_2 > k_3$$



Unit - V

Three Phase Circuits

Advantages of Three phase System :-

* The output of 3 phase machine is always greater than single phase machine of same size, which results in reduced cost.

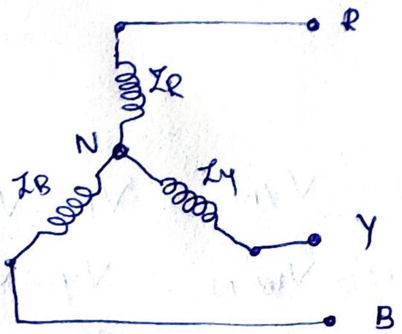
* For transmission & distribution, three phase system needs less copper than a single phase system. Hence it is very economical.

* By using three phase system, we can create rotating magnetic field, hence three phase induction motors are self starting.

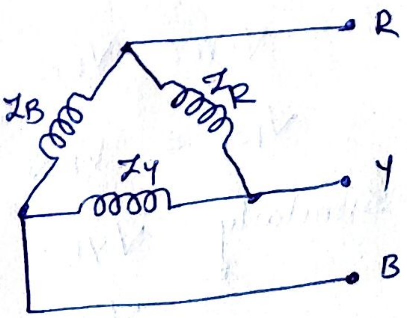
* Power factor of three phase system is better than single phase system.

Three phase supply connections :-

(i) Star connection



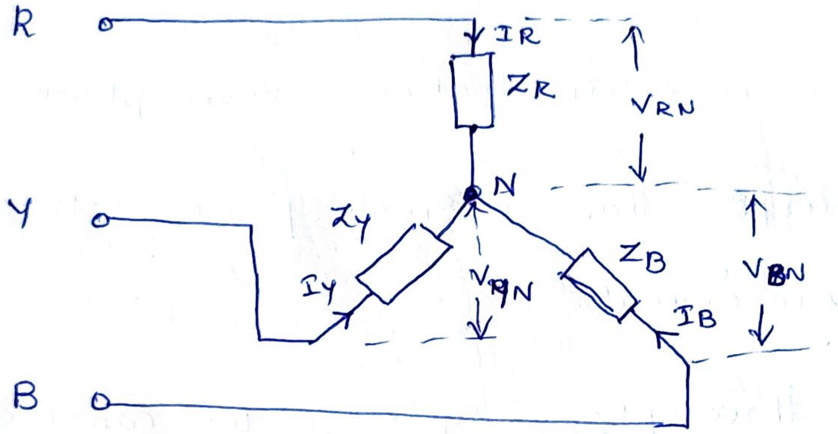
(ii) Delta connection



Balanced Load condition :-

The load is said to be balanced, when magnitude of all impedences Z_{ph1}, Z_{ph2} and Z_{ph3} are equal and phase angle of all of them are equal, irrespective of load types.

(i) star connected load :-



Let V_{RN}, V_{BN}, V_{YN} are phase voltages
I_R, I_Y, I_B are phase currents.
Z_R, Z_Y, Z_B are phase impedences.

For star connected system,

$$I_L = I_R = I_Y = I_B = I_{ph}$$

ie $\boxed{I_L = I_{ph}}$ ——— (1)

$I_L \rightarrow$ line current , $I_{ph} \rightarrow$ phase current

$$V_{ph} = V_{RN} = V_{YN} = V_{BN}$$

$$V_{RY} = V_{RN} + V_{NY} = V_{RN} - V_{YN} = V_R - V_Y$$

Similarly $V_{YB} = V_{YN} + V_{NB} = V_{YN} - V_{BN} = V_Y - V_B$

$$V_{BR} = V_{BN} + V_{NR} = V_{BN} - V_{RN} = V_B - V_R$$

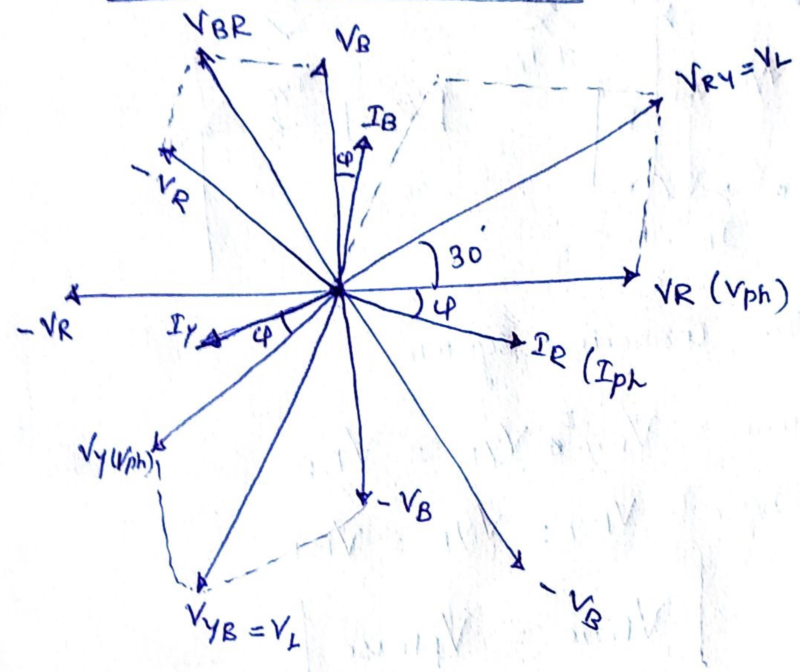
∴

phase impedence

$$\boxed{Z_{ph} = \frac{V_{ph}}{I_{ph}}} \quad \text{———} \quad (2)$$

$\varphi \rightarrow$ angle between V_{ph} & I_{ph}.

$$\boxed{\begin{aligned} Z_{ph} &= R + jX_L \\ \varphi &= \tan^{-1}\left(\frac{X_L}{R}\right) \end{aligned}} \quad \text{———} \quad (3).$$



If $Z = R - jX_C$, then

$$\boxed{\varphi = \tan^{-1}\left(\frac{-X_C}{R}\right)} \quad \text{———} \quad (4)$$

then,
$$\boxed{V_{ph} = \frac{V_{line}}{\sqrt{3}}} \quad \text{———} \quad (5)$$

Power at phase

$$P_{ph} = V_{ph}\, I_{ph} \cos\varphi$$

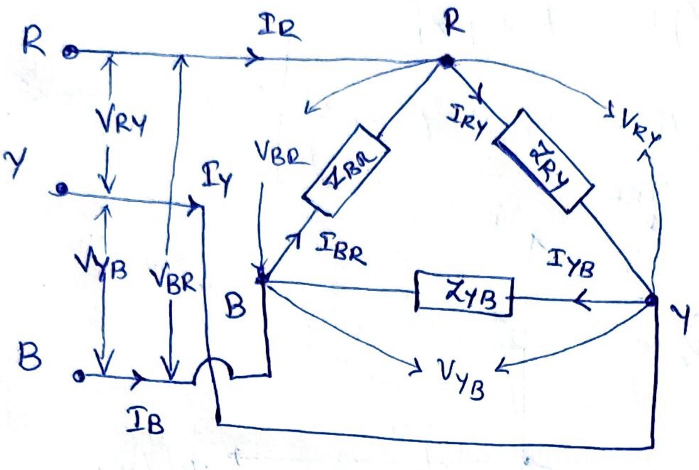
In 3φ system,

$$\boxed{\begin{aligned} P &= 3\, V_{ph}\, I_{ph} \cos\varphi \\ &\text{or} \\ P &= \sqrt{3}\, V_L\, I_L \cos\varphi \end{aligned}} \quad \text{———} \quad (6).$$

power factor $\cos\varphi = \dfrac{R}{|Z|}$.

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

(ii) <u>Delta connected Load</u>:-



For delta connected system,

$V_{line} = V_{RY} = V_{YB} = V_{BR}$

$V_{phase} = V_{RY} = V_{YB} = V_{BR}$.

$\therefore$ $\boxed{V_{line} = V_{phase}}$ ——— (1)

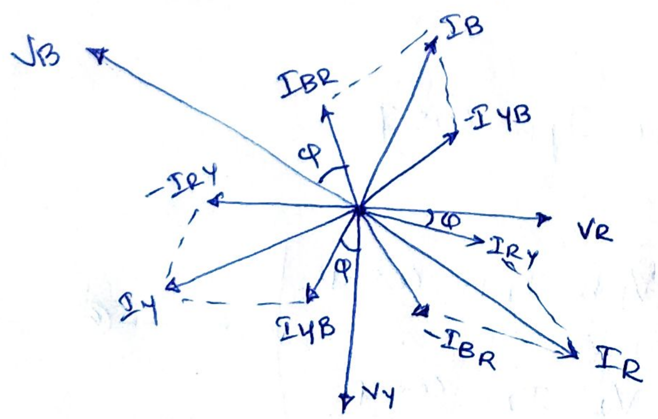
$I_{line} = I_R = I_Y = I_B$.

$I_{phase} = I_{RY} = I_{YB} = I_{BR}$

$I_R = I_{RY} - I_{BR}$

$I_Y = I_{YB} - I_{RY}$

$I_B = I_{BR} - I_{YB}$.



From phasor diagram,

$$\boxed{I_{phase} = \frac{I_{Line}}{\sqrt{3}}} \qquad (2)$$

power consumed by each phase.

$$P_{ph} = I_{ph} \times V_{ph} \cos\varphi$$

Total power in 3φ system,

$$\boxed{P = \sqrt{3}\, V_L I_L \cos\varphi} \qquad (3)$$

Reactive power,

$$\boxed{Q = \sqrt{3}\, V_L I_L \sin\varphi} \qquad (4).$$

Apparent power

$$\boxed{R = \sqrt{P^2 + Q^2}} \qquad (5)$$

<u>Unbalanced load</u>:-

when the load impedence are not equal in each phase of the available three phase of a load, then it is called to be unbalanced load.

(i) Star connected system.

$$Z_{ph1} = Z_{ph1} \angle \varphi_R = Z_R \angle \varphi_R$$

$$Z_{ph2} = Z_B \angle \varphi_B$$

$$Z_{ph3} = Z_Y \angle \varphi_Y$$

phase voltage $V_{RN} = V_{ph} \angle 0^\circ$

$V_{YN} = V_{ph} \angle -120^\circ$

$V_{BN} = V_{ph} \angle +120^\circ$

$$I_R = \frac{V_{RN}}{Z_R} = \frac{V_{ph} \angle 0^\circ}{Z_R \angle \phi_R} = \frac{V_{ph}}{Z_R} \angle -\phi_R$$

$$I_Y = \frac{V_{YN}}{Z_Y} = \frac{V_{ph} \angle -120^\circ}{Z_Y \angle \phi_Y} = \frac{V_{ph}}{Z_Y} \angle -120^\circ - \phi_Y$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{V_{ph} \angle +120}{Z_B \angle \phi_B} = \frac{V_{ph}}{Z_B} \angle +120 - \phi_B$$

$$I_N = I_R + I_Y + I_B.$$

(ii) Delta connected System,

unbalanced load impedence are

$$Z_{ph1} = Z_{ph} \angle \phi_R = Z_R \angle \phi_R$$

$$Z_{ph2} = Z_Y \angle \phi_Y$$

$$Z_{ph3} = Z_B \angle \phi_B$$

Voltages:-

$$V_{RY} = V_{YB} = V_{BR} = V_L = V_{ph}.$$

$$\boxed{V_L = V_{ph}}$$

Current:

$$I_R = \frac{V_{ph} \angle 0^\circ}{Z_R \angle \phi_R} = \frac{V_{ph}}{Z_R} \angle -\phi_R$$

$$I_Y = \frac{V_{ph} \angle -120^\circ}{Z_Y \angle \phi_Y} = \frac{V_{ph}}{Z_Y} \angle -120 - \phi_Y$$

$$I_B = \frac{V_{ph} \angle +120^\circ}{Z_B \angle \phi_B} = \frac{V_{ph}}{Z_B} \angle 120 - \phi_B.$$

$$I_{L1} = I_R - I_B, \quad I_{L2} = I_Y - I_R, \quad I_{L3} = I_B - I_Y$$

Power $P = V_{ph} I_R \cos\phi_R + V_{ph} I_Y \cos\phi_Y + V_{ph} I_B \cos\phi_B.$

Measurement of Power & Power Factor:

Power is the rate of doing work. In electrical,

(i) DC Power

$$P = V \times I \quad \text{watts.}$$

(ii) AC Power

$$P = V \times I \times \cos\varphi \quad \text{watts.}$$

(iii) In 3 phase AC power

$$P = \sqrt{3}\, V_L I_L \cos\varphi$$

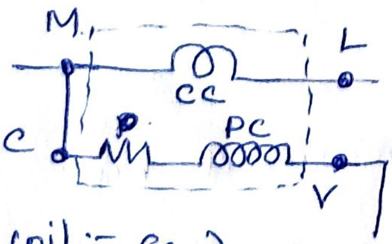
or

$$P = 3\, V_{ph} I_{ph} \cos\varphi$$

The power is measured by wattmeter.

Watt meter:-

This is a meter which measures the power consumption of a three phase or single phase load and it gives it as reading through a pointer, directly. The wattmeter will have two coils.



M → main terminal
L → Load terminal
C → common terminal
V → voltage terminal.

(i) current coil:- (CC)

The coil between M & L is called current coil, which senses the current and will always connected in series with the load. So it is called series coil.

As it is to measure the load current, current coil resistance should be as low as possible. Hence the wire used to would current coil will have large cross sectional area and wounded with less number of turns

(2) Voltage coil or Potential coil :-

This is the coil shown between terminal C & V, which senses the voltage or potential across the supply or load terminals. So it is also called as potential coil or pressure coil.

As it is always connected in parallel to the supply or load terminals, it is also called shunt coil. The resistance of the voltage coil should be maximum. So it is represented large number of coil turns and the wire used to wound this coil will be lesser cross sectional area.

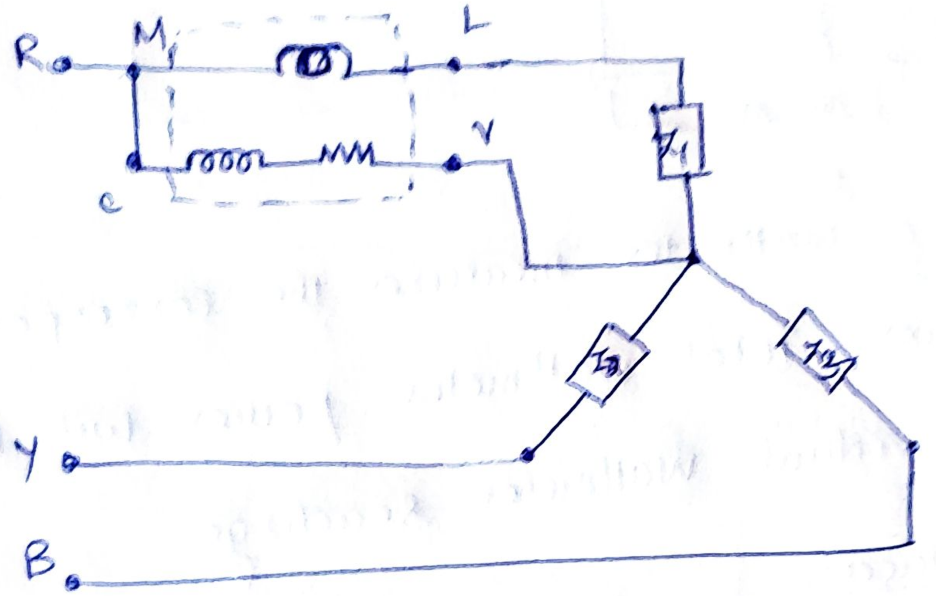
The power is measured by

$$P = V_{vc} * I_{cc} * \cos(V_{cc} \wedge I_{cc}) \text{ watts}$$

Types of Power measurement for three phase load:-

(i) one wattmeter method
(ii) two wattmeter method
(iii) three wattmeter method.

(i) one wattmeter method:-



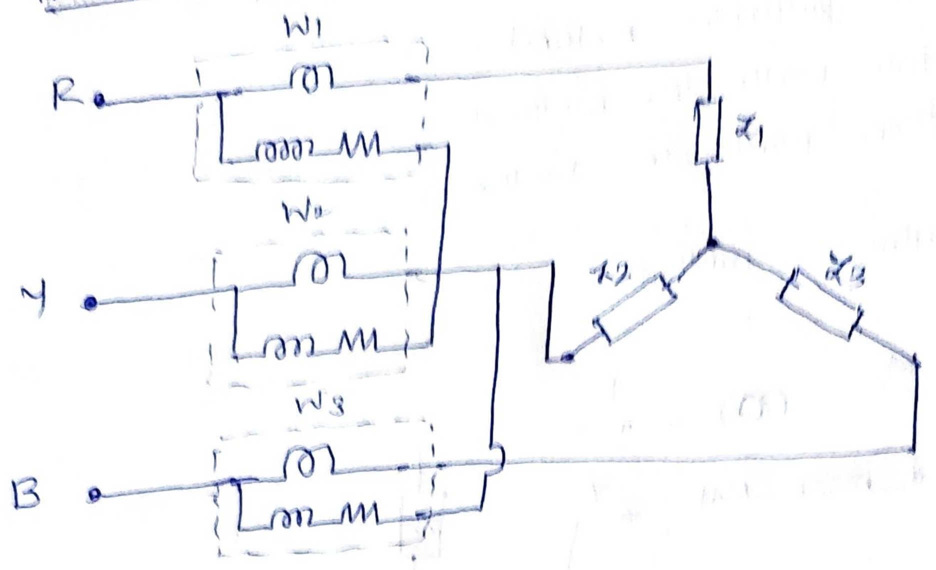
when the load is balanced, the three phase power will be same. so it get the total power, watt meter reading can be multipled by 3.

Disadvantages:-

(i) when three phase load is not balanced, then the total power consumed cannot be measured by one wattmeter method.

(ii) This method of power measurement for a delta connected load gives wrong reading and Problems

(ii) Three wattmeter method:



Each of a wattmeter measures the corresponding phase power. Total wattmeter power will be sum of individual wattmeter readings.

Disadvantages:-

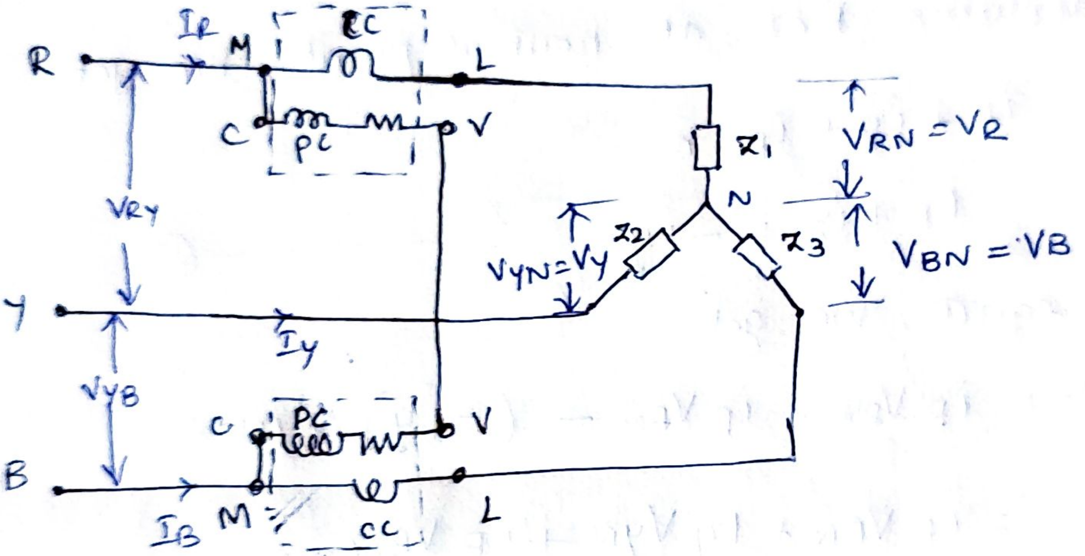
(i) It is not possible to get neutral point in all loads, In such case this method cannot be adopted to measure power.

(ii) Also delta connected load, it is not feasible to open delta connected load, to insert the current coil of the wattmeter.

Hence the best method of measuring power for any load is only using Two wattmeter method.

(iii) Two wattmeter method :-

Here two wattmeter are used to measure the three phase power. The current coil of two wattmeter are connected for R and B phases. Then the voltage of two wattmeter are connected between R-Y phases and Y-B phases.



This method is suitable for star/delta and balanced/unbalanced three phase loads. The total power can be calculated by adding two wattmeter reading.

Total power $W = W_1 + W_2$

(i) Unbalanced load:-

Let $i_R \rightarrow$ instantaneous value of current through W_1

$V_{RY} \rightarrow$ instantaneous value of voltage across W_1

$i_B \rightarrow$ instantaneous value of current through W_2

$V_{BY} \rightarrow$ instantaneous value of voltage across W_2

Now $W_{inst} = i_{cc} \times V_{pc}$

$W_1 = i_R \times V_{RY}$ & $W_2 = i_B \cdot V_{BY}$

But $V_{RY} = V_{RN} - V_{YN}$, $V_{BY} = V_{BN} - V_{YN}$

So $W_1 = i_R (V_{RN} - V_{YN})$ & $W_2 = i_B (V_{BN} - V_{YN})$

$$W_1 + W_2 = i_R V_{RN} - i_R V_{YN} + i_B V_{BN} - i_B V_{YN}$$

$$W_1 + W_2 = i_R V_{RN} + i_B V_{BN} - (i_R + i_B) V_{YN} \quad \text{——— (1)}$$

By applying KCL at neutral point; we get

$$i_R + i_Y + i_B = 0$$

$$i_R + i_B = -i_Y \quad \text{——— (2)}$$

Sub. in eqn (1), we get

$$W_1 + W_2 = i_R V_{RN} + i_B V_{BN} - (-i_Y) V_{YN}$$

$$= i_R V_{RN} + i_Y V_{YN} + i_B V_{BN}$$

$$W_1 + W_2 = P_R + P_Y + P_B$$

where P_R, P_Y & P_B → instantaneous power consumed by each phase of load.

(ii) Balanced loads:-

Let I_R, I_Y, I_B → phase current of respective phase.

V_{RN}, V_{YN}, V_{BN} → phase voltages of respective phases w.r.t N

V_{RY}, V_{YB}, V_{BR} → line voltages.

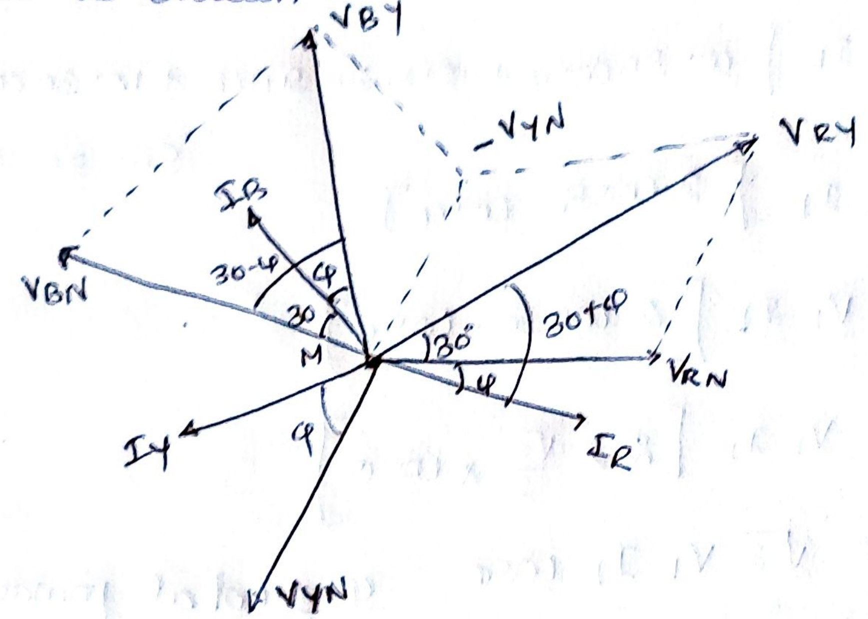
ϕ → phase angle b/w voltages and current in every phase.

Let us consider the RMS value of current and voltages to calculate the total power consumed by the load.

$$W_1 = I_R \times V_{RY} \times \cos(I_R \wedge V_{RY})$$

$$W_2 = I_B \times V_{BY} \times \cos(I_B \wedge V_{BY})$$

Assume the power factor φ to be lagging and a vector diagram is drawn.



$V_{RY} = V_{RN} - V_{YN}$, $V_{BY} = V_{BN} - V_{YN}$

phase angle $\varphi = I_R \wedge V_{RY}$

$\varphi = I_B \wedge V_{BY}$

For balanced load, $V_{RY} = V_{BY} = V_L$

$I_R = I_B = I_L = I_{ph}$.

$I_R \wedge V_{RY} = 30 + \varphi$

$I_B \wedge V_{RY} = 30 + \varphi$

$I_B \wedge V_{BY} = 30 + \varphi$

$\boxed{I_B \wedge V_{BY} = 30 + \varphi}$

$I_B \wedge V_{BY} = 30 - \varphi$

$W_1 = I_R V_{RY} \cos(30 + \varphi)$

$W_1 = W_L = V_L I_L \cos(30 + \varphi)$

$W_2 = I_B V_{BY} \cos(30-\varphi)$

$W_2 = I_L V_L \cos(30-\varphi)$.

$W_1 + W_2 = I_L V_L \cos(30+\varphi) + V_L I_L \cos(30-\varphi)$

$= V_L I_L [\cos(30+\varphi) + \cos(30-\varphi)]$

$= V_L I_L (\cos(30+\varphi) + \cos(30-\varphi))$

$= V_L I_L [\cos 30 \cos\varphi + \sin 30 \sin\varphi + \cos 30 \cos\varphi - \sin 30 \sin\varphi]$

$= V_L I_L [2 \cos 30^\circ \cos\varphi]$

$W_1 + W_2 = V_L I_L [2 \cos 30 \cdot \cos\varphi]$

$= V_L I_L [2 \times \frac{\sqrt{3}}{2} \times \cos\theta]$

$W_1 + W_2 = \sqrt{3} V_L I_L \cos\varphi$ $\quad \varphi \to$ total power

<u>power factor measurement</u>:-

$W_1 = V_L I_L \cos(30+\varphi)$

$W_2 = V_L I_L \cos(30-\varphi)$

$W_1 + W_2 = \sqrt{3} V_L I_L \cos\varphi$

$W_1 - W_2 = V_L I_L (\cos 30+\varphi) - \cos(30-\varphi)$

$= V_L I_L [\cos 30 \cos\varphi + \sin 30 \sin\varphi - \cos 30 \cos\varphi + \sin 30 \sin\varphi]$

$= V_L I_L [2 \sin 30 \sin\varphi]$

$= V_L I_L [2 \times \frac{1}{2} \times \sin\varphi$

$W_1 - W_2 = V_L I_L \sin\varphi$

Ratio $\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\varphi}{\sqrt{3} V_L I_L \cos\varphi} = \frac{\tan\varphi}{\sqrt{3}}$

$$\tan\varphi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]$$

power factor $\boxed{\cos\varphi = \cos\left[\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]\right]}$

If any of the wattmeter coil's connections is interchanges during measurements, then that reading should be negative value.

$$\tan\varphi = \frac{\sqrt{3}(W_1 - (-W_1))}{W_1 + (-W_2)}$$

$$\boxed{\tan\varphi = \frac{\sqrt{3}(W_1 + W_2)}{(W_1 - W_2)}}$$

Advantages :-

(i) power can be measured for both balanced and also unbalanced.

(ii) Two wattmeter in sufficient for the measurement of total three phase power.

(iii) Necessity of opening the delta connection of the load for inserting current coils of wattmeter is avoided.